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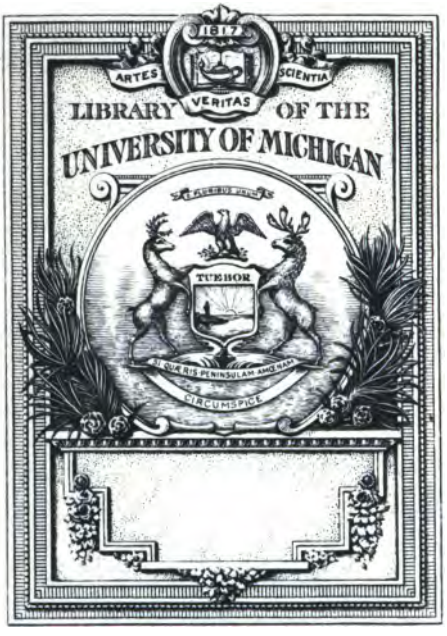
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AND

Projection of the SPHERE,

Made Eaſy.

TEACHING

The Projection of the Sphere Orthographick, and Stereographick: As alſo, Trigonometry Plain and Spherical; with plain and intelligible Reaſons for the various and moſt uſeful Methods, both in Projection and Calculation; with the Application of the whole to

Aſtronomy, *Dialling*, and *Geography*.

By HENRY WILSON.

LONDON:

Printed by *H. P.* for J. SENEX, at the *Globe* in *Salisbury-Court*; and W. TAYLOR, at the *Ship* in *Pater-Noſter-Row*. MDCCXX.

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Hist. of Reg. Bowes 9.11.31 24597

To the Worſhipful the Maſters, Wardens, and Aſſiſtants of the *Trinity-houſe* of *Deptford-Strand.*

2.3.39 M.S.N.

IT *may be thought Surpriſing, if not accounted great Preſumption, to Dedicate ſo ſmall a Treatiſe to ſo honourable and uſeful a Corporation. But my Hopes that this plain, tho' conciſe Piece of* Trigonometry, *well applied, may at leaſt contribute to the qualifying the Studious for ſome of thoſe Publick Mathematical Services, to*

which they cannot be introduced without your Examination and Approbation, hath encouraged me to caſt this my Mite into your Treaſury, which that your Worſhips may pleaſe to accept of, is the humble Deſire of

Your moſt

humble Servant,

HENRY WILSON.

TO

TO THE READER.

ALtho' the great Number of Books of all ſorts now extant in the World, might be a ſufficient Diſcouragement to any Author from offering any Thing more to the Preſs; yet the Conſideration of that common Complaint of a great Part of thoſe who have been at much Coſt and Pains to learn, viz. *that they forget a conſiderable Part of what they have learned, before they have the Opportunity to put it in Practice*, hath animated me to venture into the World with this my Manual of *Trigonometry*, in which I have endeavoured to ſerve Learners of all ſorts; the Subſtance of Projection of the Sphere and Trigonometry, being compriz'd in ſuch a moderate Volume, that it cannot be thought (by thoſe that have any Opportunity and Deſire to learn) a Difficulty to procure Money to buy it, or Time to read it; the Rules ſo clear, that it is not hard to underſtand it; and the Reaſons for them ſo plain, that it will be very eaſy to retain and remember it, and apply it to Practice as Occaſion requires.

I have first begun with the Rudiments of Geometry, so far as is necessary to the Performance of what follows.

2. Plain Trigonometry Right-angled and Oblique, in order to the right understanding of which, I have inserted some necessary Definitions and Axioms, with the Construction of Sines, Tangents, and Secants so explain'd, that the Learner may see the Reason why the Sides of the Triangle become Sines, Tangents, or Secants, to their respective Angles, which Reasons once learn'd, the Rules cannot be easily forgot.

3. Before I have entred upon Spherical Trigonometry, I have handled the Stereographick Projection of the Sphere, with general Rules for doing it upon the Plain of any great Circle. I have Instanced in five of the Principal, *viz.* The Meridian, Horizon, Equator, Ecliptick, and Prime Vertical; which, I suppose, with the Reasons there given for the several Appearances of the Projection, according to the various Positions of the Eye, will be sufficient for the Information of the Learner in all the Varieties of that Projection.

I confess I have in this Particular deviated from the Order of some Authors, *viz.* either those who treat of Spherical Trigonometry and Astronomy, without speaking any Thing of the Projection of the Sphere, or those, who in the Order of their Books speak of Spherical Trigonometry first, and Projection of the Sphere afterwards. I cannot but think it more proper and instructive to the Learner to be first taught the Projection of the Sphere, and

the

the ſeveral Circles which conſtitute thoſe Triangles, which are the Subject of Spherical Trigonometry, before they fall upon the Calculation; for it ſeems evidently more eaſy and natural to form a Proportion from what is given for the Solution of any propoſed Triangle, after it is projected in its proper Appearance, than to perform the Calculation firſt and the Projection afterward, the ſame *Data* that ſerves for one, ſerving alſo for the other, and the Proportion for Solution ſometimes depending upon the Proportions of the Sines of Sides, to the Sines of their oppoſite Angles, ſometimes requiring a Perpendicular to be let fall, *&c.* which Varieties may be beſt conceived after the Triangle is projected, and that has induced me to aſſume this Method.

In all my Problems in Aſtronomy, I have always choſen to Project upon the Plain of ſuch a great Circle, as may be moſt Commodious for Performance, and may come under the general Rule given (*pag.* 119. if in Oblique) whereas to thoſe that learn to Project only upon the Plain of the Meridian, the Projection from ſome *Data* is rendred impoſſible.

And becauſe nothing ſhould be wanting to make Spherical Operations intelligible, I have not only explained my Lord *Nepair*'s Proportion, but alſo that which I have called the *Globical Projection of the Sphere*, and therein deduced Proportions for the ſolution of all the Varieties of Right-angled Spherical Triangles (and conſequently of Oblique-angled Triangles reduced to Right-angled by Perpendiculars) from no other Principle than the ſimple Conſtruction

of

of Sines (being Perpendiculars let fall) and Tangents (being Perpendiculars raiſed from the End of Radius.) And for the better illuſtration of both, I have made uſe of this in the Caſes of Trigonometry, and of my Lord *Nepair's* Proportion in Aſtronomy, and have alſo added the Orthographick Projection of the Sphere, and the Solution of the moſt neceſſary Queſtions in Aſtronomy thereupon, which is performed by Scale and Compaſſes only, *viz.* by the Chords and Sines on the Gunter's Scale; and I have alſo inſerted the moſt material and Fundamental Problems in Dialling and Geography, and concluded with ſome uſeful Tables to find the Riſing, Setting, *&c.* of the Sun, Moon, or Planets.

Altho' I have not been ſo large in the Application of Trigonometry to Dialling and Geography, as a larger Treatiſe would admit of; yet I have given ſuch demonſtrable Reaſons for the moſt material Problems therein, as is ſufficient to incite the Learner to, and inform him in thoſe uſeful Parts of the Mathematicks; which in this, as in others of my publick Performances, is the chief Deſign of

A true Lover of the Mathematicks,

HENRY WILSON.

N. B. *I have not uſed any Symbols or Contractions, except that one in Spherical Projections* [P C Q] *which ſignifies, Draw the Primitive Circle, and Quarter it, that is, Croſs it at Right-Angles with two Diameters.*

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CHAP. I.

SECT. I.

GEOMETRICAL PROBLEMS.

PROBLEM I.

From a given Point A *in the Middle of a Line, to Erect a Perpendicular.*

WITH any Extent in your Compasses, and one Foot in *A*, make the Marks *c* and *d*; then with any greater Extent, and one Foot in *c*, make the Arch *e*, and with the same Extent, and one Foot in *d*, cross the said Arch in *e*; a Line drawn from the Intersection of the Arches in *e* to the given Point *A*, is the Perpendicular required.

Plate 1. *Fig.* 1.

PROBLEM II.

From a Point given without, to let fall a Perpendicular to a Line given.

With any Extent more than the neareſt Diſtance between the Point and the given Line, and one Foot in the given Point *X*, croſs the Line in *m* and *n*; then with the ſame Extent, (or any other greater than half the Diſtance *m n*) placing one Foot in *m* and *n*, make two Arches to croſs each other in *r*, lay a Ruler from *x* to the Croſſing at *r*, and by it draw *x b*, it is the Perpendicular required.

Plate 1. *Fig.* 2.

PROBLEM III.

From the End of a given Line, to raiſe a Perpendicular.

With any Extent, and one Foot in the End of the Line at *B*, place the other Foot any where above the Line, as at *C*, and keeping that Point at *C*, turn the other about, and croſs the Line at *D*, and turning your Compaſſes, make the Arch *E*; lay a Ruler from the Croſſing at *D* over the Point *C*, and croſs the Arch *E*, a Line drawn from the Croſſing at *E* to the Point *B*, is the Perpendicular required.

Plate 1. *Fig.* 3.

PROBLEM IV.

From a Point given over the End of a Line, to let fall a Perpendicular.

From the given Point *B* draw the Line *B D*, to cut the Line *D E* at any convenient Diſtance from

from the End *E*, whereabout the Perpendicular is to fall, divide the ſaid Line *B D* into two equal Parts at *F*, and placing one Foot in *F*, with the Extent *F B*, or *B D*, croſs the Line *D E* in *E*, the Line *B E* being drawn, is the Perpendicular required. *Plate* 1. *Fig.* 4.

PROBLEM V.

To divide a given Right Line into two Equal Parts, properly called Biſſecting *a Line.*

Let the Line be *A C*, with any Extent greater than half the Line, and one Foot in *A* make the Arch *b d*; then with one Foot in *C* croſs the ſaid Arch in *b* and *d*, a Line drawn from *b* to *d*, will biſſect the Line *A C* in *e*, as was required. *Plate* 1. *Fig.* 5.

PROBLEM VI.

At a given Diſtance, to draw a Line Parallel to a given Line.

Take the given Diſtance in your Compaſſes, and with one Foot any where towards each End of the given Line, as in *a* and *b*, ſweep the ſmall Arches *c* and *d*; a Line drawn ſo as juſt to touch the Extremity of thoſe Arches, ſhall be the Parallel required. *Plate* 1. *Fig.* 6.

PROBLEM VII.

To bring any three Points into the Circumference of a Circle, provided they are not ſituate in a Right Line.

Let the Points be *A B* and *C*, firſt with any Extent more than half *A B*, and one Foot in *A*,

make the Arch *d m e*, and with the ſame Extent, and one Foot in *B* deſcribe the Arch *d n e*, to croſs the former in *d* and *e*; then *Plate* 1. *Fig.* 7. with any Extent more than half *B C*, and one Foot in *B* and *C*, deſcribe the Arches *h i g*, and *h r g*, croſſing each other in *h* and *g*, then through the two Croſſings *h* and *g*, draw the Line *h g o*; likewiſe through *d* and *e* draw *d e o*, and where theſe Lines croſs each other, (as in *o*) is the Center of the Circle, and the Diſtance *o A*, *o B*, or *o C*, will ſweep the Circle required.

Note, By the ſame Means may be found the Center of any Circle or Arch of a Circle by aſſuming any three Points, and finding the Center as above.

PROBLEM VIII.

To lay down any given Angle at the Point B *in the Line* B C, *ſuppoſe of* 40 *Degrees.*

With the Chord of 60, (always) and one Foot in the given Point *B*, deſcribe the Arch *e g*; then take your given Degrees *Plate* 1. *Fig.* 8. 40 off the ſame Chord, and ſet from *g* to *e*, and from *B* through *e* draw *B D*, it will make an Angle of 40 Degrees at *B*, with the Line *B C*.

SECT.

SECT. II.

DEFINITIONS *and* AXIOMS.

EVERY Triangle conſiſts of three Sides, and three Angles.

In a right angled Triangle, the longeſt or ſlope Side is called the *Hipotenuſe*; the other two Sides including the right Angle, are called *Legs* or *Sides*; or ſometimes one is called the *Baſe*, and the other the *Perpendicular*, and they may be called ſo alternately according to the Poſition of the Triangle.

An Angle is commonly expreſs'd either by a ſingle Letter, as the Angle *B*, or by three Letters, as the Angle *A B C*, whereof the Middle ſignifies the Angle.

An Acute Angle is leſs than 90 Degrees, as *A C B*.

A Right Angle is 90 Degrees, as *A B C*.

An Obtuſe Angle is more than 90 Degrees, as the Angle *A C G*. *Plate* 1. *Fig.* 13.

The three Angles of any right lined Triangle added together make 180 Degrees; therefore the two Acute Angles of any right angled Triangle make always 90 Degrees, becauſe the right Angle is 90, therefore one Acute Angle of a right angled Triangle, or two Angles of an Oblique Triangle, being known, the other may be eaſily found.

When the Sine, Tangent, *&c.* of an Obtuſe Angle is required, you muſt inſtead of that

take its Complement to 180; thus the Sine of 130 Degrees is the Sine of 50 Degrees, *&c.*

Co Ar, or Complement Arithmetical, is found by subſtracting every Figure in a Sine or Tangent from 9, except that next the Right Hand, and that is to be ſubſtracted from 10. Thus the *Co Ar* of the Sine of 29 *d.* 6 ′, is 0, 31306, but may be more readily found by Inſpection in the Table of Logarithmal Sines, Tangents, and Secants, thus:

For *Co Ar* of { Sine / Tang. / Secant } Take { Sec. Compl. / Tang. Compl. / Sine Compl. } reject *Radius.*

And you have the Complement Arithmetical required.

The chief Uſe of *Co Ar* is, when *Radius* is not one of the given Terms; for then by taking the *Co Ar* of the firſt Term inſtead of the Sine or Tangent itſelf, and adding it to the other two Terms, the Sum abating *Radius*, is the fourth Term required.

A Definition of the Sines, Tangents, &c.

Plate 1. *Fig.* 10. In order to the right Underſtanding of Trigonometry, it is neceſſary to know what is a Sine, Tangent, Secant, *&c.* of which take the following Definition.

Every great Circle is ſuppoſed to contain 360 Degrees, the half Circle 180, and the Quadrant (upon which the Sines, Tangents, *&c.* are projected) 90 Degrees.

The *Radius*, Root, or Foundation of the Projection, is the Extent with which the Arch is ſwept,

ſwept, or the Semidiameter of the Circle, and is commonly ſuppoſed to be divided into 10000 Parts, as *b e*.

A right Sine is a right Line drawn from the given Degree, till it fall at right Angles upon that Semidiameter, from whoſe Extremity it is number'd as *g*, *f* is the Sine of 45 Degrees, becauſe it is a Perpendicular, let fall upon the *Radius e b*, and the 45 Degrees is number'd from *b*.

The Complement of an Arch is ſo much, as it is leſs than 90, and the Sine Complements, Tangent Complements, *&c.* are only the Sines, Tangents, *&c.* of thoſe Differences; thus *g i* equal to *f e*, is the Sine Complement of 45, and is the Sine alſo of 45 Degrees, if number'd back from 90, becauſe it is a Perpendicular let fall upon the Line *e h*, from whoſe Extremity (*h*) the 45 Degrees are number'd to *g*.

A Tangent Line is the Line *b d* continued, being Perpendicular to the *Radius e b*, and juſt touching the Circle and the Tangent of any Arch is number'd upon that Line, and limited by a right Line drawn from the Center through the Degree propoſed, and continued till it cut the Tangent Line; thus *b d* is the Tangent of 45 Degrees.

A Secant is the Line ſo drawn from the Center, till it cut the Tangent Line; thus *e d* is the Secant of 45 Degrees.

A Verſed Sine is that Part of the Diameter comprehended between the Circle and the End of the Right Sine, as *f b* is the Verſed Sine of 45 Degrees.

A Chord is a right Line drawn between the Extremities of the given Arch, as the Line *b g* is the Chord of 45 Degrees. *Note*, The Chord of any Arch is equal to twice the Sine of half the ſaid Arch.

From this Diagram well underſtood and apply'd, are deduced Rules and Proportions for ſolving the following Caſes of Trigonometry, obſerving, that as *Radius* is the Semidiameter of the Circle, ſo in the Application, whatſoever Side of a right angled Triangle you would make *Radius*, take that Side in your Compaſſes, and with one Foot in the Angle at the Baſe ſweep a Circle, then will the Angle at the Baſe become the Angle at the Center, and each Side will at once diſcover whether it be Sine Tangent, or Secant, according to the Deſcription above.

Suppoſe in the Triangle *a b c*, you would make the Baſe *a b Radius*, take *a b* in your Compaſſes, and with one Foot in *a* draw the Arch *b d*, and draw *a d* Perpendicular to *a b*, then is *a b d* a Quadrant, and by the foregoing Definitions *b c* is the Tangent, and *a c* the Secant of the Angle *b a c*.

Plate 1. *Fig.* 9.

But ſuppoſing the Triangle *e f g* to be the ſame as *a b c*, only you would make the Hipotenuſe *e g*, *Radius*, then *g f* is become the Sine, and *f e* (equal to *g i*) the Sine Complement of the Angle *e*, and theſe will be the Terms neceſſary to be made uſe of in the Solution.

Plate 1. *Fig.* 10.

All

All theſe Things rightly underſtood, all the Varieties in right lined Triangles may be reduced to theſe four *Axioms*.

AXIOM I.

In all right-angled plain Triangles, if one Side be made *Radius*, the other Sides will be Sines, Tangents, or Secants. For firſt,

If a Leg be made *Radius*, the other Leg will be the Tangent of the Angle at the Baſe, and the Hipotenuſe the Secant of the ſame Angle. *Plate* 1. *Fig.* 9.

But if the Hipotenuſe be made *Radius*, the Leg oppoſite to the Angle at the Baſe is the Sine of the ſaid Angle, and the other its Sine Complement. *Plate* 1. *Fig.* 10.

AXIOM II.

In all plain Triangles the Sides are proportionable to the Sines of their oppoſite Angles.

In right angled Triangles it is evident, if you make the Hipotenuſe *Radius*, that the Legs are the projected Sines of their oppoſite Angles, and conſequently bear the ſame Proportion to the Hipotenuſe, and between themſelves, that the Sines in the Table bear to *Radius*, and to one another. (See *Plate* 1. *Fig.* 10.).

AXIOM III.

In all Triangles, as the Sum of the Legs including any Angle is to their Difference, ſo is the Tangent of half the Sum of the unknown Angles, to the Tangent of half their Difference.

AXIOM IV.

In all right lined Triangles; as the Baſe (or longeſt Side) to the Sum of the other two Sides, ſo is the Difference of the ſaid ſhorteſt Sides to the Difference of the Segments of the Baſe; ſee this explain'd in Caſe the Sixth of Oblique Triangles.

SECT.

SECT. III.

PLAIN TRIGONOMETRY *Right-angled.*

CASE I.

THE Hipotenuſe and the Angles being given, to find a Leg.

Given { Hipotenuſe *AC* 86 } Required
{ The Angle *BAC* 36 30 } *CB.*

CONSTRUCTION.

Draw *AB* at Pleaſure, and by *Prob.* 8, make the Angle *A* 36 30, upon *AC* ſet off 86 from *A* to *C*, and by *prob.* 4. let fall the Perpendicular *CB*, then ſhall *AB* (meaſured on the ſame Scale of equal Parts, that *AC* was taken from) be 51. *Plate* 1. *Fig.* 11.

CALCULATION.

Firſt making *AC Radius*, (which is moſt eaſy, becauſe it comes under the Proportion of Sides to the Sines of their oppoſite Angles) then (by *Plate* 1. *Fig.* 10.) *CB* is the Sine of the Angle *A*, therefore

As *Radius*	——	10. 00000
To *AC* — 86	→	1. 93449
So Sine of the Angle *A* 36 30		9. 77428
To Side oppoſite, *BC* 51	—	1. 70887

But making *A B Radius*, then *A C* is the Secant, and *B C* the Tangent of the Angle *A*, (by *Plate* 1. *Fig.* 9.) therefore,

As Secant of the Angle *A* 36 30 — *Co Ar* 0. 90517
To *A C* —— 86 — 1. 93449
So Tangent of the Angle *A* 36 30 - 9. 86920

To Side *B C* —— 51 — 1. 70886

But making *B C Radius*, *A C* will (by *Plate* 1. *Fig.* 9.) be the Secant of the Angle *C*, (which is the Complement of *A*, *viz.* 53 30) and then

As Secant of the Angle *C* 53 30 - 10. 22561
To *A C* —— 86 — 1. 93449
So *Radius* 10. 00000

To Side *B C* —— 51 — 1. 70888

I ſhall not trouble the Reader with all theſe Varieties in the reſt, ſuppoſing this with a little Application ſufficient to give the Learner an Inſight into the making every Side *Radius*, and the Reaſon of the Proportions ariſing from thence.

CASE II.

Given the Angles and a Leg, to find the Hypotenuſe.

Given { The Angle *A* 40.07 } Required
{ The Leg *A B* 500 } *A C*.

CONSTRU-

CONSTRUCTION.

Make *A B* 500, and ſet off the given Angle, as in *Caſe* 1. at *A*, and draw *A C* at Pleaſure; then from *B* (by *Prob.* 3.) raiſe the Perpendicular *B C* to cut *A C* in *C*, and the Triangle is finiſhed. *plate* 1. *Fig.* 11.

CALCULATION.

Making *A C Radius*, *A B* will be the Sine of the Angle at *C*, then

As Sine of the Angle *C*	—	500	— 9. 88425
To Side *A B*	—	500	— 2. 69897
So *Radius*	—	—	10. 00000
To Side *A C* required	—	653	— 2, 81472

CASE III.

Given the Angles and a Leg, to find the other Leg.

Given { The Angle at *A* 40 0 } Required
{ The Leg *A B* -- 500 } *B C.*

CONSTRUCTION.

As in Caſe the ſecond, the ſame Thing being given.

CALCUL*A*-

CALCULATION.

Making *A B Radius*, *B C* will be the Tangent of the Angle at *A* therefore,

As *Radius*	—	—	10. 00000
To *A B* 500	—	—	2. 69897
So Tangent of the Angle *A* 40 0	—		9. 92381
To *B C* required	— 419	—	2. 62278

CASE IV.

Given the Hipotenuse and a Leg, to find an Angle.

Given { The Hipotenuse *AC* 720 } Require the
{ The Leg *A B* — 550 } Angle *A*.

CONSTRUCTION.

Make *A B* 550, and at *B* erect the Perpendicular *B C* at Pleasure, by *Prob.* 3. then with the Extent *A C* 720, and placing one Foot in *A*, cross the Perpendicular *B C* in *C*, and draw *A C*, and it is done. *Plate* 1. *Fig.* 11.

CALCULATION.

Making *A C Radius*, *A B* will be the Sine of the Angle *C*, or Sine Complement of the Angle *A*, required therefore

As *A C*	— 720	—	2. 85733
To *Radius*			10, 00000
So *A B*	— 550	—	2. 74036
To Sine Comp. of Angle *A*, 40 11			9. 88303

CASE

CASE V.

Given the Hipotenuſe and a Leg, to find the other Leg.

Given { The Hipotenuſe *AC* 720 } requir'd the
{ The Leg *AB* — 550 } Leg *BC*.

CONSTRUCTION.

As in *Caſe* 4. being the ſame *Data*.

CALCULATION.

This requires two Operations; firſt by *Caſe* the 4th, to find the Angle at *A*, which is already found to be 40. 11; then if you make *AB Radius*, *BC* is the Tangent of the Angle at *A*; but *AC* being made *Radius*, *BC* is the Sine of the ſaid Angle, and according to the latter.

As *Radius*	—		10. 00000
To *AC*	—	720	— 2. 85733
So Sine of the Angle at *A*		40 11	- 9. 80971
To the Leg *BC* required		464	-- 2. 66704

CASE VI.

The Legs given, to find an Angle.

Given { The Leg *AB* 360 } Required the An-
{ The Leg *BC* 430 } gle *A*.

plate 1.
Fig. 12.

CONSTRU-

CONSTRUCTION.

Make *AB* 360, and at *B* by *prob.* 3. erect the Perpendicular *BC*, upon which set off 430, from *B* to *C*, and draw *AC*, and 'tis done.

CALCULATION.

Making *AB Radius*, *BC* will be the Tangent of the Angle at *A*, therefore

As *AB*	—	360.	— 2. 55630
To *Radius*	—	—	10. 00000
So *BC*	—	430	— 2. 63346
To Tang. of the Angle *A* 5. 04			— 10. 07716

CASE VII.

The Legs given, to find the Hipotenuse.

Given $\left\{\begin{matrix} AB & 360 \\ BC & 430 \end{matrix}\right\}$ Required *AC*.

Conſtruction as *Caſe* the ſixth.

CALCULATION.

Find the Angle *A* by *Caſe* the ſixth, which is 50° 4′ then

Making *AC Radius*, *BC* will be the Sine of the Angle *A*, therefore

As Sine of the Angle *A*, 50° 4′			— 9. 88467
To Side *BC*	—	430	— 2. 63346
So *Radius*	—	—	10. 00000
To *AC* required	—	560. 8	— 2. 74879

There

There is another Method when the Legs are given, to find the Hipotenuſe and the Angles, *viz.* Multiply each Leg by itſelf, and add the two Products together, extract the ſquare Root of that Sum, it ſhall be the Hipotenuſe required, (*Euclid. Lib.* I. *Prop.* 47.) and having found the Hipotenuſe, take half the longer Leg, and add it to the Hipotenuſe, and ſay, As that Sum is to 86, ſo is the ſhorter Leg to its oppoſite Angle; and having found one of the acute Angles, the other is eaſily found, it being its Complement to 90 Degrees.

EXAMPLE.

In the foregoing Triangle in *Caſe* 6 and 7, *AB* 360, *BC* 430. Firſt to find *AC*.

		314500 (560	
		25	
360	430	106)645	Making the Remainder 900 a
360	430	636	Numerator to
216	129	112)900	the Root. Doubled the Hipotenuſe is 560-$\frac{200}{1120}$ which reduced is 560-$\frac{5}{28}$, which in Decimals is 560, 8 neareſt
108	172		
129600	184900		
	129600		
Sum of the Prod.	314500		

Then

Then for the Angle *C* oppoſite to the leſſer Side,

AC (to avoid Fractions) is	561
The longer Leg *BC* 430, its half is	215
	776

As 776 86 360
86

216
288

776)30960(39$\frac{456}{776}$
2328

7680
6984

696

For the leſſer Angle, *viz.* *C*, whoſe Complement 50-$\frac{456}{776}$, anſwers ſufficiently near the Truth for common Uſe.

To illuſtrate this Method of finding an Angle.

CONSTRUCTION.

Retaining ſtill the former Triangle *ABC*.

Plate 1.
Fig. 13.

Continue the longer Leg *CB* both Ways, and with the Extent *AC*, and one Foot in the leaſt Angle) deſcribe the Circle *FAD*, make *FG* equal to *FC*, erect the Perpendicular or Tangent Line *DE*, and draw *GA* continued, till it cut the ſaid Tangent *DE* in *E*.

This

This Proportion is grounded upon a Suppoſition, that the Arch *A D* differs inſenſibly from the Tangent, Line *D E*; and indeed the leſs the Angle at the Center, they ſtill differ the leſs: And even in this Caſe, where the Angle at *C* is near 40 Degrees, the Error is not material. Now, if we grant *A D* equal to *E D*, and the Diameter of a Circle to be to the Circumference, as 113 to 355, the Proportion follows thus.

In the Similar Triangles *G B A*, and *G D E*, it is as *G B* to *B A*, ſo *G D* to *D E*, (*Euclid. Lib.* VI. *Prop.* 4.) and if *D A* be equal to *D E*, it is as *G B* to *B A*, ſo *G D* to *D A*; and becauſe we are endeavouring to meaſure the Angle *B C A* by the Arch *A D*, it muſt be ſuppoſed to be an Arch of a Circle, the Whole of which contains 360 Parts or Degrees, and its half *D A F* 180, and by the above-mentioned Proportion its Diameter is $114\frac{2}{7}\frac{2}{1}$, ſuch Parts whereof the whole Circumference is 360, the half of that, *viz.* $57\frac{2}{7}\frac{1}{1}$ is *Radius*, and 3 times $57\frac{2}{7}\frac{1}{1}$ is $171\frac{6}{7}\frac{1}{1}$. Now, becauſe by Conſtruction *G D* is 3 times *Radius*, *viz.* $171\frac{6}{7}\frac{1}{1}$, which for Evenneſs of Numbers we will call 172, and *G F* is equal to *F C*, equal to *C A* the Hipotenuſe and *C B* is the longer Leg, it will be as *G B* (twice the Hipotenuſe, and once the longer Leg) to *G D* (thrice *Radius*, or) 172, ſo *B A* the ſhorter Leg, to *D E*, which is granted equal to *D A*, which (*D A*) is the Meaſure of the Angle *E* required:

And by taking half the two firſt Terms, it will be as the Hipotenuſe added to half the longer Leg is to 86, (the $\frac{1}{2}$ of 172) ſo the ſhorter Leg is to its oppoſite Angle.

SECT.

SECT. IV.

Of Oblique Plain TRIANGLES.

CASE I.

Two Angles and a Side oppoſite to one of them being given, to find the Side oppoſite to the other.

Given { The Angle *D* 35 / The Angle *E* 40 / The Side *EF* 380 } Required the Side *DF*.

Having given the Angles *D* and *E*, you have conſequently the Angle *F*, being the Complement of the other to 180

The Angle *D*	35 0	From	180 0
The Angle *E*	40 0	Subſtract	75 0
Sum	75 0	Reſt the Angle *F*	105 0

CONSTRUCTION.

Draw *DE* at Pleaſure, and by *Prob.* 8. make the Angle *E* 40 Degrees; upon the Side *EF*, ſet off 380 from *E* to *F*, and at *F* make an Angle of 105 Degrees drawing *FD*, and the Triangle is finiſhed.

Plate 1. *Fig.* 14.

CALCULA-

CALCULATION.

This is solved by *Axiom* the Second, and the Proportion is,

As Sine of the Angle *D*	35 0	Co.Ar-	0.24140
To Side oppoſite *FE*	380	---	2.57978
So Sine of the Angle *E*	400	---	9.80806
To Side oppoſite *FD* required	426	---	2.62924

CASE II.

Two Sides and an Angle oppoſite to one of them, being given, to find the Angle oppoſite to the other.

Given	The Side *DE*	500	required the Angle *F*.
	The Side *EF*	420	
	The Angle *D*	30 00	

This Caſe is ambiguous, and may have the Angle *F*, either acute or obtuſe, which therefore muſt be known before the Quantity of the Angle can be determined.

CON-

CONSTRUCTION.

Draw *F D* at Pleaſure, and by *Prob.* 8. make at *D* the given Angle 30, and draw *D E* 500; then with the Side *E F* 420, in your Compaſſes, and one Foot in *E*, croſs the Line *F D*, in *F* and *f*, one making the Angle *E F D* acute, the other obtuſe: But if it be firſt known whether the Angle *F* be acute or obtuſe, you need draw but one of the Lines *E F*, according as the Angle is, and the Triangle is finiſhed.

Plate 1. *Fig.* 15.

CALCULATION.

By *Axiom* the Second, the Proportion is,

As Side *F E* ——	400	--- Co. Ar.	7.37676
To Sine of the Angle oppoſite *D*	30 0	——	9.69897
So Side *E D* ——	500	——	2.69897
To Sine of the oppoſite Angle *F*	36 32 if acute, 143 28 if obtuſe		9.77470

CASE III.

Two Sides and an Angle oppoſite to one of them being given, to find the third Side.

Given { The Side *D E* 550, The Side *E F* 420, The Angle *D* 30 0 } required the Side *F D*.

CON-

CONSTRUCTION.

As in *Case* the Second.

CALCULATION.

Having found the Angle *F*, as in *Case* the Second, the Angle *E* is found by *Subſtraction*; if *F* be acute, the Angle *E* is 113 28; if *F* be obtuſe, the Angle at *E* is 6 32; then firſt if the Angle *F* be acute.

As Sine of the Angle *F*--36 32--Co. Ar-0.22527
To Side oppoſite *ED* -- 500 — 2.69897
To Sine of the Angle *E*--113 28 — 9.96250

To Side oppoſite *FD* — 770 — 2.88674

But if the Angle *F* be obtuſe,

As the Sine of the Angle *F*-143 28-Co. ar-0.22527
To Side oppoſite *ED* — 500 — 2.69897
So Sine of the Angle *E* — 6 32 — 9.05607

To Side oppoſite *FD* required 95 — 1.98031

CASE IV.

Two Sides and their contained Angle given, to find either of the other Angles (conſequently both.)

Given

Given { The Side *FE* 420 / The Side *FD* 770 / The Angle *F* 36,32 } To find the Angle *E*.

CONSTRUCTION.

Make *FD* 770 and by *Prob.* 8. make the Angle *F* 36,32, and draw *FE*, upon which set off 420 from *F* to *E*, and draw the Line *ED*, and 'tis done.

CALCULATION.

Plate 1. *Fig.* 16. The Performance of this depends upon *Axiom* the Third, *viz.*

As Sum of the given Sides	1190	Co.Ar-6.92446
To the Difference of the given Sides	350	2.54406
So Tang. $\frac{1}{2}$ Sum of unknown Angles	71 44	10.48138
To Tang. $\frac{1}{2}$ their Difference	41 43	9.94990
Half Difference added to the half Sum	113 27	the Angle *E*.
Subtracted from the half Sum	30 1	the Angle *D*.

CASE V.

Given two Sides, and their contained Angle, to find a third Side.

Given

Given { The Side *FD* 770, The Side *FE* 420, The Angle *F* 36.32 } required the Side *ED*.

CONSTRUCTION.

The ſame *Data* and Conſtruction as the foregoing Caſe.

CALCULATION.

Find the Angle as in *Caſe* the 4th, where the Angle *D* is 30° 1′ then, as in *Caſe* the firſt.

As Sine of the Angle *D* — 30.1—Co. ar. 0.30081
To Side oppoſite *EF* — 420 — 2.62324
So Sine of the Angle *F* — 36 32 — 9.77472

To Side oppoſite *ED* required 500 — 2.69877

CASE VI.

Three Sides given to find an Angle.

Given { The Side *FD* 770, The Side *FE* 420, The Side *ED* 500 } required the Angle *F*.

CONSTRUCTION.

Plate 1. *Fig.* 17. Make *FD* 770, then from the ſame Scale take in your Compaſſes 420, and with one Foot in *F*, make the Arch *E*; then with 500 in your Compaſſes, and one Foot in *D*, croſs the Arch *E* in *E*, draw *FE* and *D E*, and 'tis done.

CALCULATION.

The Solution of this depends upon the 4th *Axiom*, 3 ſas before you can proceed to the Operation, the moſt common and eaſy Way, you muſt firſt ſuppoſe the oblique Triangle to be divided into two right angled ones, by the Perpendicular let fall from *E* to *a*; and to find the Segments *F a*, and *a D*, the Rule by *Axiom* 4, is,

As the Baſe	— 770	Co. ar. 7.11351
To the Sum of the Sides	920	— 2.96378
So the Difference of the Sides	80	— 1.90309
To the Difference of the Baſes	95	— 1.98038

The Difference *D a* 95, taken from the whole Baſe *FD* 770; half the Remainder, *viz. F a* 337.5 is the Baſe of the right angled Triangle *F a E*, in which to find the Angle *F*, making *F E* Radius, *F a* will be the Sine Complement of the Angle *E*; therefore,

As

As *F E*	---	420	Co. ar.	7.37676
To Radius	---	---	---	10.00000
So is *F a*	---	337.5	---	2.52827
To S. C. of the Angle F. required	36 32	---		9.90503

Three Sides of any right lin'd Triangle, in one Sum given, and the Angles alſo given to lay it down, and find the Sides ſeverally.

Given { Sum of the Sides 80 } { The Angles 55 0, 46 0, 79 0 } to find a Side.

Draw *a b* 80. Make the Angle *b a c* equal to half one of the given Angles; as ſuppoſe, half of 55.0, *viz.* 27 30, and draw *a c*, then make the Angle *a b e* half 46 0, *viz.* 23.0, and draw *b c* to cut *a c* in *c*, from the Middle of the Lines *a c* and *c b* draw the Perpendiculars *h d* and *g e*, and where they cut the Line *a b*, as in *d* and *e*, make Marks, to which draw the Lines *c d* and *c e*, then *c d e* is the Triangle required. *Plate* 1. *Fig.* 18.

Then in the Triangle *a c b* you have given the Angle *a c b* 129 30 (the Angle *a c d* being half the Angle *c d e Eucl. Lib.* 1. *Prop.* 32. and *e c b* is half *c e d by the ſame*,) and there is alſo given the Side *a b* 80, and the Angle *b* 23:0 to find the Side *a c*, which being found, you have

given in the Triangle *a c d* the Angle *c d a* 125 0, it being the Complement of *c d e* to 180, *Euclid. Lib.* 1. *Prop.* 13. and the Side oppoſite *a c*, and the Angle *c a d* 27 30 to find its oppoſite Side *c d*; and when that is found, you have in the Triangle *d c e*, one Side, and all the Angles to find the other Sides ſeverally.

In the right angled Triangle *a b c*, there is given *a b* 96, and *b c* and *c a* in one Sum 220, to find the Sides and Angles ſeverally.

Plate 1. *Fig.* 19. Draw *b a* 96, and perpendicular thereto at *b*, draw *b d* 220, and draw *a d*, which biſſect in *e*, and draw the Perpendicular *e c*, to cut *b d* in *c*, and draw *a c*, then is *a b c* the Triangle required. Then for Calculation in the Triangle *a b d*, you have given *b d* 220, and *a b* 96, to find the Angle at *d*, which is equal to the Angle *c a e* (becauſe their oppoſite Sides are equal.) Likewiſe the Angle *d a b* is the Complement of *a d b* to 90; then from the Angle *d a b* ſubtract the Angle *e a c*, there remains the Angle *c a b*, whoſe Complement is the Angle *a c b*, then in the ſaid Triangle *a b c*, there is given the Baſe *a b*, and the Angles *a* and *b*, to find the Hipotenuſe by *Caſe* 2. or the Perpendicular by *Caſe* 3. but I ſhall leave the Operation to the Learner's Practice.

CHAP.

CHAP. II.

SECT. I.

Of the PROJECTION *of the* SPHERE.

THERE are ſeveral Ways of Projecting the Sphere, *viz.* Stereographick, Orthographick, and Gnomonical. I ſhall only ſpeak of the Stereographick at this time, as being moſt uſeful in Trigonometrical Operations, and ſhall refer the other to their proper Places afterwards.

Stereographick Projection of the Sphere, is the Projection of the Sphere or Globe, upon a Plain, paſſing through the Center thereof; the Eye being ſuppoſed to be placed upon the Superficies or Surface of the Globe, perpendicular to the Center of the ſaid Plain, or in the Pole of that great Circle, upon which

the Projection is to be made, and on that Side of the Sphere oppoſite to the Side to be projected; this Projection thus made, ſhall truly repreſent all the Appearances of that half of the Sphere, oppoſite to that, in whoſe Pole the Eye is placed, as may be illuſtrated by the following Similitude.

Suppoſe a Sphere or Globe were made of Glaſs, or any tranſparent Metal, on which were deſcribed all the Points and Circles of the Sphere (as *Zenith*, *Nadir*, *Equator*, *Ecliptick*, &c.) in black Lines that might be diſcerned through the Globe; and admit there were a Glaſs Plain paſſing through the Center of the Globe, cutting it in two equal Parts in that great Circle, upon whoſe Plain the Projection is to be made; as for Inſtance, ſuppoſe I were to project the Weſtern Hemiſphere upon the Plain of the Meridian, having a tranſparent Globe, prepared as before deſcribed, and the tranſparent Plain to paſs through the Center of the Sphere, and cut it in two Parts in the Meridian (the great Circle upon which the Projection is to be made) if the Eye be ſuppoſed to be placed in the Eaſt Point of the Horizon, upon the Surface of the Sphere; and obſerve, where the Meridians, Azimuths, &c. on the Weſt Side of the Sphere cuts the Plain paſſing through the Middle of the Sphere: Theſe Circles upon that Plain ſhall truly repreſent the Projection of the Weſtern Hemiſphere upon the Plain of the Meridian; for the Projection of the Sphere, of what kind ſoever, is no more but the

the Repreſentation of thoſe Circles and Points upon a Plain; which are imagined to be deſcribed in the Sphere, or upon the Celeſtial or Terreſtrial Globe.

The imaginary Points, are 1. The North and South Poles of the World, or of the Equator. 2. The Poles of the Ecliptick. 3. The Equinoctial Points *Aries* and *Libra*, where the Ecliptick cuts the Equator. 4. The *Zenith* and *Nadir*.

The Circles of the Sphere, are either greater or leſſer Circles. The greater Circles are thoſe that divide the Sphere itſelf in two equal Parts; as 1. The Horizon which divides that Part of the Sphere viſible to us, from that Part that is inviſible. 2. The Meridian, which divides the Eaſt half from the Weſt. 3. The Equator, which divides the North Half from the South. 4. The Ecliptick. 5. The Prime Vertical, *&c.* of all which ſee a more particular Deſcription afterwards.

The leſſer Circles divide the Sphere in two unequal Parts, as the Almicanthar or Parallels of Altitude, and alſo the Parallels of Declination, ſo called, becauſe they are Parallel to the Equator, *&c.*

The Sphere may be Stereographically projected upon the Plain of any great Circle; I have for the Learner's Information, inſtanced in five Varieties, having projected the Sphere.

Upon the Plain of the { Meridian, Fig. 1. / Horizon, Fig. 2. / Equator, Fig. 3. / Ecliptick, Fig. 4. / Prime Vertical, Fig. 5. } *Plate* 5.

Upon the Plain of whatſoever great Circle the Sphere be projected, all the Circles in the Sphere will be Circles in the Plain of the Projection, and if continued to perfect Circles, will anſwer all the Repreſentations and Conditions of Meaſuring, *&c.* without the primitive Circle, as well as within it; and alſo the Angles made by the Interſection of two great Circles on the Surface of the Sphere, are equal to the Angles made by their Repreſentatives on the Plain of the Projection.

In any Projection, that Circle upon whoſe Plain the Sphere is projected, is called the primitive Circle, and is always a perfect Circle deſcribed about the whole Projection, as the Circle *Z H N O*, Fig. 1. or the Circle *N W S E*, Fig. 2. *&c.* in *Plate* 2.

All the reſt of the great Circles in any Projection, are either right Circles, or oblique Circles.

A right Circle appears to the Eye to be a right Line, cutting the primitive Circle at oppoſite Points, and paſſing through the Center thereof, thereby dividing the Superficies of it in two equal Parts, as in *Fig.* 1. the Lines *P a S* and *Æ a Q* and *E a C* are right Circles.

An

An oblique Circle cuts the Primitive in oppoſite Points, but does not paſs through the Center of it, as in *Fig.* 1. the Circles *P* ⊙ *S*, and *Z* ⊙ *N*, are oblique Circles.

A Parallel, or leſſer Circle, neither cuts the Primitive at oppoſite Points, nor (except accidentally) paſſes through its Center, as the Circle *b* ⊙ *d*, *Fig.* 1. is a leſſer Circle, *viz.* a Parallel of Altitude, or an Almicanthar.

SECT. II.

How to project and meaſure any of the aforeſaid Circles Stereographically, which may properly be called Spherical Geometry.

PROBLEM I.

How to find the pole of any great Circle.

Note, THE Pole of a great Circle is a Point every Way 90 Degrees diſtant from its Periphery, for every great Circle contains 360 Degrees, and if two great Circles cut each other at right Angles, (*viz.* one Perpendicular to the other) there will be 180 Degrees of one Circle on each Side of the other, and the half of that, *viz.* 90 Degrees number'd either Way from the

Periphery of any great Circle, finds its Pole; thus the Zenith and Nadir are the Poles of the Horizon, and the Poles of the World are the Poles of the Equator, *&c.*

CASE I.

To find the Pole of the Primitive Circle.

Find its Center, and that is its Pole required.

CASE II.

To find the Pole of a right Circle, suppose of the right Circle *Æ a Q*.

Cross the Diameter *Æ a Q* at right Angles, with the Diameter *p a S*, the two Points *p* and *S* at a Quadrant's Distance from *Æ* and *Q* are the Poles of the Right Circle, *Æ a Q* required.

Plate 2. *Fig.* 1.

CASE III.

To find the Pole of an oblique Circle, suppose of the Circle *Z ⊙ N*.

Note, The Pole of an oblique Circle is always in that Diameter that passes through its Center, and cuts it in the Middle at right Angles: Thus the Pole of the oblique Circle *Z ⊙ N* is in the Diameter *H a* O, and is found thus.

Through *a* the Center of the primitive, and O the Center of the oblique Circle, (which in this Case falls without the primitive) draw the

the Diameter *H a O*; lay a Ruler from *Z* to *R*, and it cuts the primitive in *h*; from *h* set off 90 Degrees to *K*, and then a Ruler laid from *K* to *Z* cuts the Diameter *H a* O in *L*; the Pole of the Circle *Z* ⊙ *N* required, and (*Fig.* 2.) the Pole of the oblique Circle *w q e*, is in the Point *x*, *v*.

PROBLEM II.

To find the Center of any given oblique Circle, if it be known what Angle it makes with the primitive. (Plate 2. Fig. 1.)

To find the Center of the oblique Circle *Z R N*, which makes an Angle with the primitive Circle at *Z* of 50°.

VARIETY I.

With the Tangent of the given Angle, and one Foot in *a*, the Center of the primitive, the other Foot extended along the Diameter *H a* O, (continued if need be) will reach to the Point *o*, the Center required.

Orthus.

VARIETY II.

The Point *K* being found by *Prob.* I. *Case* 3. the Extent *N K*, set from *K* to *y*, a Ruler laid from *z* to *y*, cuts the Diameter *H a* O in *o*, the Center required.

VARIETY III.

Or it may be found without having the Diameter *H a* O, thus —— with the Secant of the given Angle at the primitive, and one Foot in *N*, ſweep the ſmall Arch *o*, with the ſame Extent; and one Foot in *Z*, croſs the ſaid Arch in *o*, that Croſſing is the Center required.

PROBLEM III.

To project any given Angle. (Plate 2. Fig. 1.)

CASE I.

To project any given Angle (ſuppoſe of 23 30) at the Center of the primitive.

Having drawn the primitive, and croſſed it at right Angles, with the Diameters *P a S*, and *Æ a Q*, ſet the Chord of the given Angle 23 30, from *Æ* to *E*, and through *E* and *a*, draw the Diameter *E a* C, then ſhall *Æ a E*, or *B a* ⊙, be the Angle required.

CASE II.

To make any given Angle at the Periphery of the primitive, as ſuppoſe I would make at the Point *Z* an Angle with the primitive of 50° 0′.

Find the Center O, by *Prob.* 2. *Variety* 2 or 3, then with one Foot in *o*, and the other in *z* or *n* 3 draw the Circle Z ⊙ *N*, and 'tis done;

done; and the Angle HZR is the Angle required.

CASE III.

To make an oblique Circle, to make at the primitive any given Angle with another oblique Circle.

Having given the Circle $W q E$, making an Angle with the primitive 35 20, it is required to draw another oblique Circle, to make at W an Angle of 23 30, with the oblique Circle $W q E$. *Plate 2. Fig. 2.*

Lay a Ruler from W to q, it ſhall cut the primitive in b; then ſet off the Chord of the given Angle 23 30, from b to a. A Ruler laid from a to W, cuts the Diameter $N S$ in C, then from E through C to W, draw an oblique Circle, and 'tis done.

Thus you may make any Angle, though you do not know what Angle the given Circle $W q E$ makes with the primitive; but if it be known what Angle the given oblique Circle makes with the primitive, you may find the Center of the required oblique Circle by *prob.* 2. *Var.* 3. As here the Circle $W q E$ is ſuppoſed to make an Angle of 35 20, with the primitive at W, and the Circle required to be drawn is to make at W an Angle of 23 30, which added to 35 20, the Sum is 58 50, the Angle which the Circle $W c E$ is to make with the Primitive; therefore with the Secant of 58 50, and one Foot in W, and E, find the Center by *prob.* 2. *Var.* 3. which falls without.

CASE

CASE IV.

To project any given Angle within the Periphery, but not at the Center.

VARIETY I.

To draw an oblique Circle through any given Point in a Right Circle, to make with the ſaid Right Circle any given Angle.

It is required to draw an oblique Circle through the Point ⊙ in the Right Circle $Q \odot R$, to make an Angle at ⊙ of 50.00, with the ſaid Right Circle.

Plate 2. Fig. 4.

Find the Poles of the Right Circle by *Prob. 1. Caſe 2.* which here are the Points *g* and *h*.

Lay a Scale from *g* to ⊙, it cuts the primitive in *K*, the Arch *Q K* meaſured on the Chords, is 20 12, therefore take the Tangent of 20 12, and ſet from *o* (the Center of the primitive) to *c*, and draw *c n* perpendicular to *Q R*, ſo is *c n* continued a Line, in which the Centers of all oblique Circles fall that paſs through the Point ⊙ ---. Then with the Chord of 60, and one Foot in ⊙, deſcribe the Arch *a b*, upon which from *a* ſet off the Chord of the Complement of the Angle required, which is 40 0, (becauſe the required Angle is 50 0,) to *b*, a Scale laid from ⊙ to *b*, cuts the perpendicular *m n* in *x*, then upon *x* as a Center with the Diſtance *x* ⊙, draw the oblique Circle *S* ⊙ *T*, and 'tis done; and the Angle *S* ⊙ *Q* is the Angle 50 00 required.

VARIETY

VARIETY II.

Through any given Point in an oblique Circle, to draw another oblique Circle, to make with the Former any given Angle, *Plate* 1. *Fig.* 4.

Through the Point ⊙, in the oblique Circle *S* ⊙ *T*, it is required to draw another oblique Circle, to make an Angle of 27 19, with the ſaid given Circle.

Through the Point ⊙ and the Center *q*, draw the Right Circle *Q q R*, then meaſure the Angle *Q* ⊙ *S*, by *Prob.* 10. of *Spherical Geometry*, which will be found to be 50 00; to which add the Angle required to be made 27 19, the Sum 77 19 is the Angle that the oblique Circle is to make with the aforeſaid Right Circle *Q a R*, and is projected as in the laſt *Variety*, its Center being thereby found to be in the Point *r*, and *y* ⊙ *z* is the Circle required.

Note, If the Nature of the Queſtion had required the Circle to be projected to lye nearer the Right Circle, than the given oblique Circle; you muſt have ſubſtracted the Angle required to be made from the Angle, that the given oblique Circle makes with the Right Circle; the Remainder is the Angle that the required oblique Circle is to make with the Right Circle.

EXAMPLE.

It is required through the Point ⊙, in the oblique Circle *y* ⊙ *z*, to draw another oblique Circle, to make with the former an Angle of 27 19. *Plate* 1. *Fig.* 4.

Having

Having drawn the Right Circle $Q \odot R$, through the Point $\odot$, and meaſuring the Angle $y \odot Q$, it will be found to be 77 19; from which ſubſtract the Angle required to be made, *viz.* 27 19, the Remainder 50 00 is the Angle, that the Circle to be projected is to make with the Right Circle, and may be perform'd by the foregoing Rules.

PROBLEM IV.

To draw a great Circle through any given Point to make with the primitive any given Angle.

Through the Point *n*, to deſcribe a Circle, to make with the primitive an Angle of 35 20.

Plate 2. Fig. 2.

With the Tangent of 35 20, and one Foot in the Center *y*, deſcribe the Arch *x*; then with the Secant of 35 20; and one Foot in the given Point *n*, croſs the ſaid Arch at *x*, that Croſſing is the Center of the Circle required; then with the Diſtance *x n*, draw the Circle *w n e*, it makes at *w*, the Angle 35 20, with the primitive, as was required.

Note, When you ſweep with the Tangent of the given Angle, and one Foot in the Center, you may carry the Sweep round, and then it will be eaſier to find where the Secant cuts it.

PRO-

PROBLEM V.

Through any two Points, (as ⊙ and Z) to draw a great Circle.

Through either Point, as ſuppoſe the Point ⊙, and the Center of the primitive, draw the Diameter *Q* ⊙ *a R* continued, and croſs it at Right Angles, with the Diameter *h a g*. *Plate 2. Fig. 5.*

From the Point ⊙ draw a Line to the Extremity of the ſecond Diameter, as the Line ⊙ *g*.

At the End of that Line, *viz.* at *g* erect the Perpendicular *g y*, to cut the Diameter *Q R* in *y*, then through the Points *y z* and ⊙, draw a great Circle, and 'tis done.

Or becauſe there will always be 180 of the half Tangents, between *y* and ⊙ having drawn the Diameters *g h*, and *Q d* continued, produce *g* ⊙ to *b*, lay a Ruler from *b* to *a*, it will cut the primitive in *c*; then a Ruler laid from *g* to *c*, cuts *Q R* in *y*, the third Point through which the propoſed Circle is to paſs.

Or meaſure *a* ⊙ from the Beginning of the half Tangents, which being found, ſubſtract it from 180; the Remainder taken from the half Tangents will reach from *a* to *y*, the third Point required, and the Points *y z* ⊙ is brought into an Arch of a Circle, by the ſeventh Geotrical Problem.

PRO-

PROBLEM VI.

To draw a great Circle perpendicular to a given great Circle.

Note, Any great Circle, whether Right or Oblique, drawn through the Poles of another great Circle, is always perpendicular to that great Circle, through whose Poles it is drawn.

CASE I.

To draw a great Circle perpendicular to the primitive.

Draw a Diameter through the Center of the primitive, and 'tis done; thus *Z N* and *H O* are perpendicular to the primitive.

Plate 2. Fig. 1.

CASE II.

To draw a Right Circle perpendicular to a Right Circle given.

It is only to croſs the given Right Circle at Right Angles with another Right Circle, and then the Extremities of the one are the Poles of the other; thus the Right Circle *H a o* is perpendicular to the Right Circle *Z a N*.

CASE III.

To draw an oblique Circle perpendicular to a Right Circle.

Find

Find the Poles of the given Right Circle, by *prob.* 1. *Case* 2. then through these two Poles draw an oblique Circle, and 'tis done; — thus the oblique Circle *Z R n* is perpendicular to the Right Circle *H a* Q, because cutting its Poles *z* and *n*.

CASE IV.

To draw an oblique Circle perpendicular to a given oblique Circle; suppose to the oblique Circle *w q E*. Find the Pole *r* by *prob.* 1. and through *r* draw the oblique Circle *m o z*, it is the Circle required. *Plate* 2. *Fig.* 2.

Note, If the Point *o* be given, draw an oblique Circle through the Points *o* and *r*, by *Prob.* 5. Or if the Angle with the primitive be given, draw the Circle through the given Point *r*, by *Prob.* 4.

PROBLEM VII.

To project a parallel Circle.

In this *Problem* are three Varieties.

I. To lay down a Parallel to the primitive Circle, suppose 20° 12′ distant from it, or 69° 48′ from its Pole, with the half Tangent of 69 48 its Distance from the Pole of the primitive, and one Foot in the said Pole at *o*, sweep the Circle ⊙ *v w*, it is the Parallel required. *plate* 2. *Fig.* 4.

II. To

II. To lay down a Parallel to a Right Circle at 20 12 Diſtance from it; ſet 20 12 of the Chords from *g* to *m*, and from *h* to *o*; then with the Tangent Complement of 20 12, *viz.* 69 48, and one Foot in *m*, (or with the Secant of 69 48, and one Foot in *a*) find the Point *P*, then with the Extent *P m*, and one Foot in *P*, draw the Parallel *m n o*, it is the Parallel required.

Plate 2. Fig. 5.

III. To draw a Circle parallel to the oblique Circle *a b c*, at 40 12 Diſtance from it, or 49 48 from its Pole.

Find the Pole of the oblique Circle, by *Prob.* 1. (which will alſo be the Pole of the Parallel) which here is at *d*, ſet off the half Tangent of 49 48, from *d* to *r* and *s*, the Diſtance *r s*, biſſected in *e*, gives the Center of the Parallel, and with the Diſtance *e r*, or *e s*, deſcribe the Circle *r s t*, it is the Parallel required.

Plate 2. Fig. 3.

Or becauſe the half Tangents are projected, by laying a Ruler from *a* to the equal Diviſions of the Semi-circle *a n c*, and where theſe cut the Diameter, *i n* continued are the half Tangents; therefore ſetting 49 48 of the Chords from *K*, (the Point where a Ruler laid from *a* over the Pole *d*, cuts the primitive) both Ways to *m* and *z*, a Ruler laid from *a* to *m*, and *z* cuts the Diameter *i n* in *r* and *s*, which biſſected in *e*, finds the Center required.

PRO-

PROBLEM VIII.

To meaſure an Arch of a Circle.

General Rule.

In all great Circles, whether right or oblique, (for the primitive Meaſures on the Chords without farther Application) a Ruler laid from the Poles of the Circle, to be meaſured, over the Extremities of the Arch to be meaſured, and Marks made where the Ruler ſo laid, cuts the primitive, the Diſtance between thoſe Marks meaſured on the Chords, gives the Quantity of the Arch required.

EXAMPLE.

1. It is required to meaſure the Arch *P z* of the primitive Circle *z e n H.*

Take the Extent *z P* in your Compaſſes, and apply it to the Line of Chords, it gives 35 20, the Quantity of the Arch required. *Plate 2. Fig. 1.*

2. To meaſure the Arch *a L* of the Right Circle *H a e.*

From *z* the Pole of the Right Circle *H a e*, lay a Scale over *L*, it will cut the primitive in *K*; the Arch of the primitive *n K* meaſured upon the Chords, is 50^{d} the Arch *a L* required.

3. To

3. To measure the Arch of an oblique Circle.

It is required to measure the Arch $q\ n$ of the oblique Circle $w\ q\ E$.

Find the Pole r by *Prob.* 1. then from r lay a Ruler over n, it will cut the primitive in a, the Arch $s\ a$ measured upon the Chords, is 64° 0′, the Quantity of the Arch $q\ n$ required.

Plate 2. Fig. 2.

PROBLEM IX.

To set off any Number of Degrees upon a given Circle from a given Point.

This is only the Reverse of *Prob.* 8. for laying a Ruler from the Pole of the given Circle over the given Point make a Mark, where it cuts the primitive, set the Chord of the given Number of Degrees from that Mark upon the primitive, and from the Extremity of that Extent to the Pole of the given Circle, lay a Ruler, and where that cuts the given Circle, determines the Degree required.

EXAMPLE.

It is required to set off 64 Degrees from q toward E, upon the oblique Circle $w\ q\ E$.

From the Pole r, lay a Ruler over q, it cuts the primitive in s, from s set off 64 of the Chords to a, a Ruler laid from the Pole r to a, cuts the oblique Circle $w\ q\ E$ in n, the Arch $q\ n$ is 64 Degrees, the Quantity required to be set off from q.

Plate 2. Fig. 2.

By

By the ſame Method of laying a Ruler from the Pole to the primitive, you may ſet off any Number of Degrees upon a Right Circle, as in *Fig.* 1. If I would ſet off 50 Degrees from *a*, upon the Right Circle *H a o*; if you lay a Ruler from *z* over *a*, it cuts the primitive in *n*, ſet the Chord of 50 from *n* to *K*, a Ruler laid from *z* to *K*, cuts *H a o* in *L*; then is *a L* 50 Degrees of the Right Circle required, or it may be ſet off by the Scale of half Tangents.

The primitive is always meaſured upon the Chords.

PROBLEM X.

To meaſure a Spherical Angle.

General Rules.

An Arch of a Circle paſſing between the two Sides, containing the Angle, and at a Quadrant's Diſtance from it, (of which conſequently the Angular Point muſt be the Pole) meaſured by *Prob.* 8. is the Meaſure of the Angle required. —— Or

The Diſtance between the Poles of the Circles, including the Angle, is the Quantity of the Angle required; and theſe General Rules hold true in all Caſes, whether Primitive, Circle, Right, or Oblique.

In this *Problem* are four *Varieties*. *Plate* 2. *Fig.* 1.

I. A Right Circle with the primitive makes always Right Angles, as *z n*, and *H o*, with the primitive *z H n o*.

II. A

II. A Right Circle with a Right Circle, and here becauſe the Poles of all Right Circles are in the primitive, the Diſtance between their Poles, is only meaſured thereon.

I deſire to know the Quantity of the Angle *Æ a H*. The Pole of *Æ Q* is *P*, and the Pole of *H O* is *z*, and *z P* meaſured on the Chords is 35 20, the Angle *Æ a H* required.

Plate 2. Fig. 1.

III. A Right Circle with an oblique Circle, as ſuppoſe it is required to know what Angle the Right Circle *N S* makes with the oblique Circle *Z r m*.

Find the Pole of the Right Circle *E*, and of the oblique Circle *t*, by *Prob.* 1. A Ruler laid from the Angle *r*, to the Pole of the oblique Circle *t*, will cut the primitive in *l*, and *E* being the Pole of the Right Circle; the Arch *l E* meaſured on the Chords 50 00, is the Quantity of the Angle *r* required.

Plate 2. Fig. 2.

IV. An oblique Circle with the primitive.

The Angle *q w s* is required to be meaſured.

The Pole of the primitive is the Center *y*; the Pole of the oblique Circle *w q E* is at *r*. A Ruler laid from the Angular Point *w* to *y*, cuts the primitive in *E*, and from *w* to *r*, cuts it in *d*; the Arch *d E* meaſured on the Chords is 35 20, the Quantity of the Angle *q w s* required.

Plate 2. Fig. 2.

V. An oblique Circle with an oblique Circle; the Angle $Z \odot P$ is required.

Find their Poles L and x, by *Prob.* 1. A Ruler laid from $\odot$ over L, and x cuts the primitive in y and w; the Arch $y\,w$ meaſured on the Chords, gives the Angle required. *Plate* 2. *Fig.* 1.

Note, All theſe *Varieties* come under the firſt General Rules; only we have given an Inſtance in each, to make them more intelligible.

CHAP. III.

Of Spherical Trigonometry.

SECT. I.

DEFINITIONS.

Plate 2. Fig. 1.

THE ſeveral Circles and Points given, or required, being projected as before is taught; any three great Circles interſecting each other, the Arches comprehended between the three Points of Interſection, do conſtitute a Spherical Triangle; and ſuch Triangles, conſtituted of Arches, of any three great Circles of

of the Sphere, are the Subject of Practical Spherical Trigonometry: As in *Fig.* 1. the Circles *E* ⊙ *a* ♋, and ♎ *b a* ♑, and *P* ⊙ *b S*, do cut each other in the three Points *a b* ⊙; the three Arches *a b*, *b* ⊙, and ⊙ *a*, making up the Triangle *a b* ⊙.

Of Spherical Triangles, are three Sorts, *viz.* Right-angled, Oblique-angled, and Quadrantal.

A Right-angled Triangle, hath one Angle 90 Degrees: As the Triangle *a b* ⊙, Right-angled at *b*, *Fig.* 1.

A Quadrantal Triangle, hath one Side 90 Degrees: As the Triangle *a* ⊙ *z*, *Fig.* 1. whose Quadrantal Side is *a z*.

An Oblique Triangle, hath no Side, nor Angle 90 Degrees: As the Triangle *Z P* ⊙, *Fig.* 1.

Of an Oblique Triangle, some of the Angles are acute, and some obtuse.

An Acute-angle, is less than 90 Degrees: As the Angle *Z P* ⊙, *Fig.* 1.

An Obtuse-angle, is more than 90 Degrees: As the Angle *P Z* ⊙, *Fig.* 1.

☞ *NOTE*, When two Letters are express'd, they signify the Side contain'd between them; but when three are express'd, they signify the Angle at the middle-most Letter: Thus the Angle *P Z* ⊙, signifies the Angle at *Z*, in the Triangle *P Z* ⊙, *&c.*

SECT. II.

SOLUTIONS.

In Spherical Triangles obſerve theſe Solutions.

1. THAT the Perpendicular and Baſe of a Right-angled Spherical Triangle, are of the ſame Affections with their oppoſite Angles, *viz.* More or Leſs than a Quadrant, as their oppoſite Angles are.

2. If the Legs (and conſequently the Angles) of a Right-angled Triangle, be both More, or both Leſs than a Quadrant, the Hipothenuſe is leſs than a Quadrant; but if of different Affections, *viz.* one More, and the other Leſs, the Hipothenuſe is more than a Quadrant.

3. Conſequently, if the Hipothenuſe be leſs than a Quadrant, the Legs are of the ſame Affection; if more, they are different.

4. If the Hipothenuſe be leſs than a Quadrant, each Leg ſhall be of the ſame Affection with its adjacent Angle; but if more, they are of different Affections.

5. Conſe-

5. Conſequently if a Leg be of the ſame Affection with its adjacent Angle, the Hipothenuſe is leſs than a Quadrant, and the contrary.

6. If you deſcribe a great Circle at 90 Degrees Diſtance from any Angle, the Arch of this Circle intercepted between the Legs of the Angle, is the Meaſure of the Angle.

7. The Poles of the three Sides of any Triangle, conſtitute another Triangle; for the three Angles of this Supplemental Triangle, are made by the mutual Interſection of the Circles, of which the Sides are formed in the Triangle, whoſe Sides are given that are equal thereunto.

NOTE, In this Caſe you uſe the Complement of the greateſt Side, or the greateſt Angle, to a Semicircle.

8. In an Oblique Spherical Triangle, if the Angles at the Baſe be both of the ſame Affection, the Perpendicular let fall from the Vertical Angle, falls within the Triangle; but if different, it falls without.

9. When an Oblique Spherical Triangle is to be divided into two Right-angled Ones, by a Perpendicular, you muſt let the Perpendicular fall from the End of a given Side, and oppoſite to a given Angle.

10. There are twenty eight Cases in Spherical Triangles, *viz.* Sixteen in Right-angled, and Twelve in Oblique; all which may be solved by the Axioms and Rules following.

SECT. III.

AXIOMS *and* CONCLUSIONS.

AXIOM I.

THE Sine of Middle-part, and Radius, are reciprocally proportionable to the Tangents of the Extreams-conjunct, and to the Sine-complements of the Extreams-disjunct:

That is, As Radius to Tangent of one Extream-conjunct; so the Tangent of the other Extream-conjunct, to the Sine of the Middle-part.

And as Radius to Sine-complement of one Extream-disjunct; so the Sine-complement of the other Ext. Disj. to Sine of Middle-part.

But if Middle-part be given, and one of the Extreams required, the Proportion may be inverted to bring always the required Term into the last Place thus:

As Sine-complement of the given Extream-disjunct, is to Radius; so is the Sine of Middle-part, to the Sine-complement of the Extream-

tream-disjunct required: And the like in Extreams-conjunct, as a little Practice will make plain.

AXIOM II.

In all Spherical Triangles, the Sines of the Sides are proportionable to the Sines of their oppoſite Angles, and the contrary.

Theſe two Axioms, with the Help of the following Concluſions, or Rules, are ſufficient for ſolving all the twenty eight Caſes of Spherical Triangles.

1. As the Sine of the Baſe to Radius; ſo the Tangent of the Perpendicular, to the Tangent of the Angle, at the Baſe of a Right-angled Triangle.

2. In all Right-angled Triangles, as the Sine of the Hipothenuſe to Radius; ſo is the Sine of the Perpendicular, to the Sine of the Angle at the Baſe.

Theſe two Concluſions are thus demonſtrated by Mr. *Norwood*; For the Firſt,
Let *a d B* be a Spherical Triangle, Right-angled at *B*; then is *a d* the Sine of the Hipothenuſe, *a B* the Sine of the Baſe, *a M* is Radius, and *M L* the Tangent of the Angle at *a*, and *B k* the Tangent of the Perpendicular *B d*: Then; *Plate* 3. *Fig.* 3.

As *a B* to *a M*; ſo *B k* to *M L* by *Eucl. Lib. 6. Prop.* 4.

For the Second, *a d* is the Sine of the Hipothenuſe, and *a I* is Radius, *d c* is the Sine of the Perpendicular *d B*, and *H I* is the Sine of the Angle at *a*: Then;

As *a d* to *a I*; ſo *d e* to *I H* by *Eucl. Lib. 6. Prop.* 4.

3. The Sines of the Angles ſubtended by the Perpendiculars, are directly proportional to the Sines of the Perpendiculars ſubtending.

4. If two Perpendiculars ſubtend equal Angles, the Sines of the Perpendiculars are directly proportional to the Sines of the Hipothenuſes.

☞ 5. The Co-ſines of the Angles at the Baſe, are directly proportional to the Sines of the Vertical Angles.

☞ 6. The Co-ſines of the Segments of the Baſe, are directly proportionable to the Co-ſines of the Hipothenuſes.

☞ 7. The Sines of the Baſes, are reciprocally proportional to the Tangents of the Angles at the Baſe.

☞ 8.

☞ 8. The Tangents of the Sides, are reciprocally proportional to the Co-fines of the Vertical Angles.

9. The Tangents of the Baſes, are proportional to the Tangents of the Vertical Angles.

10. Having let fall the Perpendicular, as before, it will be as the Sine of the Sum of the Hipothenuſes, to the Sine of their Difference; ſo the Co-tangent of half the Sum of the Vertical Angles, to the Tangent of half their Difference; and ſo the Co-tangent of half their Difference, to the Tangent of half their Sum.

11. As Tangent of half the Sum of the Sides or Hipothenuſes, to Tangent of half their Difference; ſo Tangent of half the Sum of the Angles at the Baſe, to Tangent of half their Difference.

☞ 12. As Tangent of half the Baſe, to Tangent of half the Sum of the Sides; ſo Tangent of half the Difference of the Sides, to Tangent of half the Difference of the Segments of the Baſe; which half Difference, added to the half Sum, gives the greater Baſe, and ſubſtracted, gives the leſſer.

Theſe two Axioms, and twelve Concluſions, I thought fit to inſert for Variety: Nevertheleſs, all the twenty eight Caſes of Spherical

rical Triangles may be ſolved by the two Axioms, and the five Concluſions marked thus ☞, *viz.* The 5, 6, 7, 8 and 12; for the ſixteen Caſes in Right-angled Triangles, are ſolved by the firſt Axiom: The firſt and fourth Caſes in Oblique, are ſolved by the ſecond Axiom: The Second, Third, Fifth, Sixth, Seventh, Eighth, Ninth and Tenth in Oblique, are ſolved by Axiom the firſt, with the Help of the Concluſions above-mention'd, *viz.* The 5, 6, 7 and 8; provided you obſerve in your firſt Operation, to find ſuch a Part, which when found, the other Part of the Thing required, which is to be added to, or ſubſtracted from the firſt Part found, may come under one of the four Concluſions above-mention'd; and the eleventh Caſe where three Sides are given, and the twelfth Caſe where the three Angles are turned into Sides, are both ſolved by Concluſion the twelfth, and Axiom the firſt, only in letting fall a Perpendicular, in this Caſe, it muſt always fall from the greateſt Angle, upon the greateſt Side; and ſo the leſſer Sides become Hipothenuſes to two Right-angled Triangles, which being given, and the Baſes found by Concluſion 12, you have enough to find with Eaſe any other Part of either Triangle; but for the Sake of thoſe that have learned another Way, and are not willing to leave it, I ſhall infert the following Axiom, which we may call,

AXIOM

AXIOM III.

As the Rect-angle of the Sines of the Sides containing the Angle required, to the Square of Radius; ſo is the Rect-angle of the Sines of half the Sum of the three Sides, and of the Difference of the Side oppoſite there-from, to the Square of the Sine-complement of half the Angle ſought.

From hence we may deduce this Rule for Operation.

Add the three Sides together, and from their half Sum, ſubſtract the Side oppoſite to the Angle required.

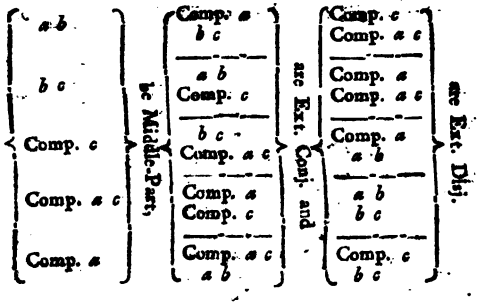
Then to the Complement Arithmetical of the Logarithm-ſines, of the containing Sides, add the Logarithm-ſines of that half Sum, and Remainder; half the Total of theſe four Logarithms is the Sine-complement of half the Angle required.

For the better underſtanding of my Lord *Nepair's* Univerſal Proportion, laid down in the firſt Axiom, you muſt underſtand, That in every Triangle there are ſix Parts, *viz.* Three Sides and three Angles; but omiting the Right-angle, there are but Five, *viz.* The three Sides, and the two Acute-angles, of which the two Legs, including the Right-angle, and the Complements of the Hipothenuſe, and two *Plate* 3. *Fig.* 2.

Acute-angles; are called the five Circular-parts, of which there are always Two given, to find a Third; and of theſe Three, *viz.* Two given, and One required; One is always Middle-part, and the other Two are either Extreams-conjunct, or Extreams-disjunct; for if the three Parts lie together, the Middle-moſt is Middle-part, and the other Two are Extreams-conjunct, as in the Figure annex'd: Suppoſe the Angles *a* and *c*, were given to find the Hipothenuſe *a c*, than *a c* is Middle-part, and the other two Parts, *viz.* the Angles *a* and *c*, are Extreams-conjunct; and by Axiom the firſt, the Proportion ſhould be as Radius to Tangent, of one Extream-conjunct; ſo the Tangent of the other Extream-conjunct, to the Sine of Middle-part: But becauſe in this Caſe, the Queſtion falls amongſt theſe Parts that are noted by their Complements, you muſt inſtead of Tangent, uſe Tangent-complement, and inſtead of Sine, uſe Sine-complement (or in Caſe they had been Extreams-disjunct, you muſt inſtead of Sine-complement, have uſed Sine) and then the Proportion is as Radius to the Tangent-complement of the Angle at *a*, one Extream-conjunct; ſo the Tangent-complement of the Angle at *c*, the other Extream-conjunct, to Sine-complement of *a c* the Middle-part.

But if Middle-part be given, you muſt alter the Proportion; for the required Term muſt always be the laſt in the Proportion: As ſuppoſe the Angle at *a*, and the Hipothenuſe *a c*, were given to find the Angle at *c*; here *a c* is Middle-

Middle-part, as before; but becauſe it is an Extream-conjunct that is required, the Proportion is as Tangent-complement of *a* (the given Extream-conjunct) to Radius; ſo the Sine-complement of *a c* (the Middle-part) to Tangent-comp. of *c*, the required Extream Conj. And thus however the three Parts that enter the Queſtion be diſpos'd of, you may always find ſuch an One to make Middle part, as that the other Two will either be both Extreams-conjunct, or Extreams-disjunct, *viz.* If the three Parts lie together, the Middle is Middle-part, and the other two Extream-conjuncts; but if one lie remote, it is Middle-part, and the other Extream-disjuncts, as in the following Table.



Quadrantal Triangles, are alſo wrought by the ſame Laws of Right angled Triangles, allowing the Quadrantal-ſide in one, to be the ſame as the Right-angle in the other; and as in Right-angled Triangles, the Legs, or Sides, next the Right-angle, and the Complements of the

the Hipothenuſe, and Acute-angles, are the five Circular-parts; ſo in Quadrantal Triangle, the Quadrantal-ſide is omited, and the two Angles, adjacent to the Quadrantal-Side, and the Complements of the other two Sides, and of the Angle oppoſite to it, make the five Circular-parts, and come under the Rules of the Catholick Proportion: Of which ſee an Example in *Prob.* 24. of Aſtronomy, performed by Right-angled Triangles; and in *Prob.* 15. of Oblique: But for the better Underſtanding hereof, Suppoſe the Triangle *a b c*, to be a Quadrantal Triangle, *viz.* The Side *a b* 90 Degrees, then is the Angle *a*, and the Angle *b*, with Complement of the Side *a c*, and Complement of the Angle *c*, and Complement of the Side *e b*, the five Circular-parts; and ſo in all others.

Plate 3. *Fig.* 1.

SECT.

SECT. IV.

The Globical Projection of the Sphere.

THERE is alſo another Way to find the Proportions for ſolving the ſixteen Caſes of Right-angled Spherical Triangles, by the or Globical Projection of the Sphere: Of which take this brief Account (*Plate* 3. *Fig.* 4.)

With the Chord of 90 Degrees, and one Foot of the Compaſſes any where, as in *A*, draw the Circle *O H K d*; then with the ſame Extent, and one Foot any where in the Periphery of that Circle, as in *O*, draw the Circle *A R F G*, ſet the Chord of the Angle at *A* (as ſuppoſe 23. 30.) from *O* to *Q* (although in this you need not be exact; for you may find the Proportion as well, if you make the Angle *A* at Pleaſure) then with the Chord of 90, and one Foot in *Q*, ſweep the Circle *A E F N*; and becauſe the Triangle is to be Right-angled at *B*, take the Chord of 90, and ſetting one Foot at ſome convenient Diſtance from *A* (as ſuppoſe at *B*) with the other Foot find the Point *m*, in the ſame Circle *A E F N*, upon which the Right-angle is to be, and upon *m*, as a Center, deſcribe the Circle *B C P G H*, then is the Triangle *A B C* finiſhed; then with the ſame Extent, and one Foot in *C*, draw

draw the Circle *O I K E*, and then is the Projection fitted for our intended Uſe.

This being done, obſerve theſe four Rules; First, What Parts of your Triangle is given: As ſuppoſe the Hipothenuſe *A C*, and the Angle at *A*, to find the Leg *A B*; I mark the Side *A C*, and alſo the Angle at *A*, with a Daſh thus ', and the Required-leg with this Mark °; then becauſe *A C* is given, *C R* is alſo given, being its Complement to 90; becauſe *A R* is 90 of the Chords, and *R S* equal to *A C*, is alſo given; all which mark for given, as you ſee, and you may proceed round the Circle if you will, to mark *S F*, *F G*, *G U*, *U L*, and *L A*.

The Angle *A* being given, its Meaſure *R d*, being an Arch 90 Degrees diſtant from it, is alſo given; and *R Q* its Complement, *Q O* equal to *R D*, and *O N* its Complement, *&c.* which mark as before; then becauſe *A B* is required, *B D* its Complement is alſo required, as alſo *A m* its Complement, and *M N* equal to *A B*, and *N T* its Complement; all which Mark required, as in the Figure.

2. Then obſerve in which Manſion your Proportion falls, which is ſure to be in that Part of the Projection of which the Triangle is the Corner, drawn here in double Lines, and bounded by the Letters *A B d R S P O Q Q*. — A Manſion is a Right-angled-triangle, conſiſting of a Baſe, a Hipothenuſe, and two Perpendiculars; as the Diviſion *A C R d B* or *B C Q R d*, &c. And when Perſons

ſons are well skill'd in this kind of Projection, they may, inſtead of the whole Projection, only draw that Part which I have bounded with double Lines; nor need you be careful to have all the Circles, or Parts therein exactly proportionable; but may draw it with a Pen, at Pleaſure, holding to the ſame Form; only always obſerve to place the Triangle at the bottom Corner, as here you ſee; and to find your Proportion, obſerve in which Manſion you have three Things given, and One required, which is here in the Manſion *Q R d B C*, Right-angled at *R* and *d*, the Side *Q d* being the Baſe, and *Q B* the Hipothenuſe, and *C R* and *d B* are Perpendiculars; and here you have *Q R* given, and alſo *Q d* Radius, which is always given; and you have *C R* alſo given, to find *B D* required, in which the Proportion is as in Plain-triangles, *viz.* As the Short-baſe *Q R*, to the Long-baſe *Q d*, or Radius; ſo the Short-perpendicular *R C*, to the Long-perpendicular *B d*.

3. But to know whether the Proportions lie in Sines, or Tangents, or both, you may obſerve as in Plain-trigonometry, when the Baſe is made Radius, the Perpendicular is a Tangent; but when the Hipothenuſe is made Radius, the Perpendicular is a Sine; ſo here, when the Proportion lies in the Baſe and Perpendiculars, the Long-baſe is Radius, and the Short-baſe a Sine, and the two Perpendiculars are Tangents; but when the Proportion lies in the Hipothenuſe and Perpendiculars,

lars, the Long-hipothenuſe is Radius, and the Short-Hipothenuſe is a Sine, and the Perpendiculars are Sines.

Fourthly and Laſtly, To know whether your Proportion lies in Sines and Tangents, or in their Complements, obſerve whether the Parts of the Manſion, where the Proportions lies, be the Parts of the Triangle given, or required, or the Complements of thoſe Parts: As in this Caſe in the Manſion $Q\ d\ B$, the Proportion is as $Q\ R$ to $Q\ d$; ſo $R\ C$ to $d\ E$, by Rule the ſecond; but by Rule the third, $Q\ R$ is a Sine, $Q\ d$ is Radius, and $R\ c$ and $d\ B$ are Tangents; and by Rule the fourth, $Q\ R$ is the Complement of the Angle at A (becauſe it is the Complement of $d\ R$ the Meaſure of it) and $C\ R$ is the Complement of $A\ C$, and $B\ d$ is the Complement of $A\ B$, therefore the Proportion is

As ($Q\ R$) the Sine-com. of the Angle at A,
To ($Q\ d$) Radius;
So ($R\ C$) the Tangent-compl. of $A\ C$,
To ($B\ d$) the Tangent-compl. of $A\ B$.
} P. 3. F. 4.

I ſhall for the Learner's further Satisfaction anſwer all the ſixteen Caſes of Right-angled Spherical Triangles, by this Method; and when I come to the Application of them to Queſtions in Aſtronomy, I ſhall anſwer the Triangles by the Proportions found by Middle-parts, and Extreams-conjunct and disjunct, that nothing may be omited that is neceſſary for

for the Reader's Inſtruction; nor yet, that I may not needleſſy repeat that in Aſtronomy which has already been ſufficiently taught in the Trigonometrical-part.

SECT. V.

Of Spherical Trigonometry Right-angled.

I Shall here only inſert the Operations by the Globical Projection of the Sphere, and leave the Stereographick-projection of them, till we come to the Application of Aſtronomy, it being the more Practical-part of Spherical-trigonometry, and aiming at Brevity, I ſhall endeavour to avoid Prolixity on the one Hand, as well as Deficiency on the other.

CASE *the firſt.*

Given the Hipothenuſe, and an Angle to find the Leg adjacent to the Given-angle.

Given { The Hip. — 60°-0′ / Angle *a* — 23—30 } Required *a b.*

Here

Plate 4. *Fig.* 1. Here the Proportion lies in the Manſion *b g d*, Right-angled at *d*; and hence by the foregoing Rules the Proportion is,

As *g e* Sine-comp. of *a*, — —	23:30 —	9.96240
To *g d* Radius; — — —	90:00 —	10.00000
So *c e* Tang. Comp. of *a c*, —	60:00 —	9.76144
To *b d* Tang. Comp. *a b*. —	57:48 —	9.79904

CASE *the ſecond.*

Given the Hipothenuſe, and an Angle to find the Leg oppoſite to the given Angle.

Given { *a c* — 60 00 / *a* — 23 30 } Required *b c*.

Plate 4. *Fig.* 2. Here the Proportion lies in the Manſion *a d e*, and the like in the Manſion *h k e*; and either Way it is.

As *a e*, or *h e* Radius, — —	90:00 —	10.00000
To *a c*, or *h g*, the Sine of *a c*;	60:00 :	9.93753
So *d e*, or *e k*, the Sine of *a*, —	23:30 :	9.60070
To *b c*, or *g i*, the Sine of *b c*.	20:12 :	9.53823

CASE

CASE *the Third.*

Given the Hipothenuſe, and an Angle, to find the other Angle.

Given { *a c* 60 00 / *a* — 23 30 } Required *c*. *Plate* 4. *Fig.* 3.

The Proportion lyes in the Manſion *c k i*, and and *is*,

As *c e* Sine Comp. of *a c*	60 00	9.69897
To *c k* Radius	90.00	10.00000
So *g e* Tang. Comp. of *a*	23.30	10.36170
To *k i* Tang. of *c*	77.44	10.66273

CASE *the Fourth.*

Given the the Hipothenuſe and a Leg, to find the Angle between them.

Given { *a c* — 60 00 / *a l* — 57 48 } Required *a*. *Plate* 4. *Fig.* 4.

The Proportion lies in the Manſion g *d b*, and it is,

As *d b* Tang. Comp. of *a l*	57.48	9.79915
To *c e* Tang. Comp. of *a e*	60.00	9.76144
So *g d* Radius	90.00	10.00000
To *g e* Sine Comp. of *a*	23.32	9.96229

CASE

CASE *the Fifth.*

Given the Hipothenuſe and a Leg, to find the Angle oppoſite to the given Leg.

Plate 4. *Fig.* 5. Given $\left\{\begin{matrix} a\,c & 60:00 \\ b\,c & 20:12 \end{matrix}\right\}$ Required *c*.

The Proportion lyes in the Manſion *a d e*, or *h k e*, and it is,

As *a c* Sine of *a c*	——	60:00	9.93753
To *a e* Radius	——	90:00	10.00000
So *c b* Sine of *c b*	——	20:12	9.53819
To *d e* Sine of *a*	——	23:30	9.60066

CASE *the Sixth.*

Given the Hipothenuſe and a Leg, to find the other Leg.

Given $\left\{\begin{matrix} a\,c & 60\ 00 \\ a\,b & 57\ 48 \end{matrix}\right\}$ Required *b c*.

Plate 4. *Fig.* 6. The Proportion lyes in the Manſion *g d b*, and will be,

As

As *d b* Sine Comp. of *a b* — 57 48— 9.72662
To *c e* Sine Comp. of *a c* — 60 oc— 9.69897
So *g b* Radius ——— 90 : oc—10.00000

To *g c* Sine Comp. of *b c* 20:13— 9.97235

Case *the Seventh.*

Given an Angle and a Leg adjacent, to find the Leg oppofite to given Angle.

Given $\left\{\begin{matrix} a — 23\ 30 \\ a\,b\ 57.48 \end{matrix}\right\}$ Reqd. *b c*. *Plate* 4. *Fig.* 7.

The Proportion lyes in the Manfion *a d e*, and it is,

As *a d* Radius —— ——— 90 : oc—10.00000
To *a b* Sine of *a b* ——— 57 : 48— 9.92747
So *d e* Tang. of *a* ——— 23 : 30— 9.63830

To *b c* Tang. of *b c* ——— 20 : 12— 9.56577

Case *the Eighth.*

Given an Angle and a Leg adjacent, to find the Angle oppofite to the given Leg.

Given $\left\{\begin{matrix} c — 77 : 44 \\ b\,c — 20 : 12 \end{matrix}\right\}$ Required *a*. *Plate* 4. *Fig.* 8.

The Proportion lyes in the Manfion *c k i*, and it is,

As

As *c i* Radius — — — 90 : 00 — 10.00000
To *i k* Sine of *c* — — 77 : 44 — 9.98997
So *c g* Sine Comp. *c b* — 20 : 12 — 9.97243

To *g e* Sine Comp. of *a* — 23 : 30 — 9.96240

CASE *the Ninth.*

Given an Angle and a Leg adjacent, to find the Hipothenuſe.

Plate 4. *Fig.* 9. Given { *c* — 77 44 / *b c* — 20 : 12 } Required *a c*

The Proportion lies in the Manſion *h k e*, and it is,

As *h i* Sine Comp. of *c* — 77 : 44 — 9.32728
To *i g* Tang. of *b c* — 20 : 12 — 9.56576
So *h k* Radius — — — 90 : 00 — 10.00000

To *k e* the Tang. of *a c* — 60 : 00 — 10.23848

CASE *the Tenth.*

Given an Angle and a Leg oppoſite, to find the Leg adjacent to the given Angle.

Plate 4. *Fig.* 10. Given { *a* — 23 30 / *b c* 20 12 } Required *a b*.

The

The Proportion lyes in the Manſion *a d e*, and it is,

As *d e* Tang. of *a*	23 : 30	—	9.63830
To *c b* Tang. of *c b*	20 : 12	—	9.56576
So *a d* Radius	90 : 00	—	10.00000
To *a b* Sine of *a b*	57 . 48	—	9.92746

CASE *the Eleventh.*

Given an Angle and a Leg oppoſite, to find the Angle adjacent to the given Leg.

Given { *a* — 23 30 } Required *c*. *Plate* 4. *Fig.* 11.
{ *b c* — 20 12 }

The Proportion lies in the Manſion *c k i*, and it is,

As *c g* Sine Comp. of *b c*	20 : 12	—	9.97243
To *g e* Sine Comp. of *a*	23 : 30	—	9.96240
So *c i* Radius	90 : 00	—	10.00000
To *i k* Sine of *c*	77 : 44	—	9.98997

CASE *the Twelfth.*

Given an Angle and a Leg oppoſite, to find the Hipothenuſe.

plate 4. *Fig.* 12. Given $\left\{\begin{matrix} a & 23 \cdot 30 \\ b\,c & 20 \cdot 12 \end{matrix}\right\}$ Req. *a c.*

The Proportion lies in the Manſion *a d e*, and it is,

As *d e* Sine of *a*	23 : 30	9.60070
To *b c* Sine of *b c*	20 : 12	9.53819
So *a e* Radius	90 : 00	10.00000
To *a c* Sine of *a c*	60 : 00	9.93749

CASE *the Thirteenth.*

Given the Legs, to find an Angle.

Plate 4. *Fig.* 13. Given $\left\{\begin{matrix} a\,b & 57\ 48 \\ b\,c & 20\ 12 \end{matrix}\right\}$ Required *a.*

The Proportion lies in the Manſion *a d e*, and it is,

As *a b* Sine of *a b*	57 . 48	9.92747
To *a d* Radius	90 . 00	10.00000
So *b c* Tang. of *b c*	20 . 12	9.56576
To *d e* Tang. of *a*	23 . 30	9.63829

CASE *the Fourteenth.*

Given the Legs to find the Hipothenuſe.

Plate

Given $\left\{\begin{matrix} a\,b & 57\ 48 \\ b\,c & 20\ 12 \end{matrix}\right\}$ Required $a\,c$. *Plate 4. Fig. 14.*

The Proportion lyes in the Manſion $g\,d\,b$, and it is,

As $g\,b$ Radius ——— 90.00 — 10.00000
To $g\,c$ Sine Comp. of $b\,c$ —— 20.12 — 9.97243
So $b\,d$ Sine Comp. of $a\,b$ —— 57.48 — 9.72663

To $c\,e$ Sine Comp. of $a\,c$ — 60.00 — 9.69906

CASE *the Fifteenth.*

Given the Angles, to find a Leg.

Given $\left\{\begin{matrix} a & 23\ 30 \\ c & 77\ 44 \end{matrix}\right\}$ Required $b\,c$. *Plate 4. Fig. 15.*

The Proportion lyes in the Manſion $c\,k\,i$, and it is,

As $i\,k$ Sine of c —— 77.44 — 9.98997
To $g\,e$ Sine Comp. of a —— 23.30 — 9.96240
So $c\,i$ Radius ——— 90.00 — 10.00000

To $c\,g$ Sine Comp. of $b\,c$ —— 20.12 — 9.97243

CASE *the Sixteenth.*

Given the Angles, to find the Hipothenuſe:

Plate 4. *Fig.* 16. Given $\left\{\begin{matrix} a \text{——} 23.30 \\ c \text{——} 77.44 \end{matrix}\right\}$ Required $a\,c$.

The Proportion lyes in the Manſion $c\,k\,i$, and it is,

As $i\,k$ Tang. of c ———	77 44	——10.66269
To $g\,e$ Tang. Comp. of a	23.30	——10.36170
So $c\,k$ Radius ———	90.00	——10.00000
To $c\,e$ Sine Comp. of $a\,c$	60.00	—— 9.69901

SECT. VI.

Spherical Trigonometry Oblique.

CASE *the Firſt.*

GIVEN two Sides and an Angle oppoſite to one of them, to find the Angle oppoſite to the other, if it be known whether the required Angle be acute or obtuſe.

Plate 4. *Fig.* 17. Given $\left\{\begin{matrix} a\,b & 42\ 12 \\ b\,c & 35\ 20 \\ c & 32\ 12 \end{matrix}\right\}$ Required a.

By

By Axiom the Second.

As Sine of *a b*	42 . 12	co. ar. 0.17282
To Sine of *c*	32 . 12	9.72663
So Sine of *b c*	35 . 20	9.76218
To Sine of *a*	27 . 19	9.66163

The Angle at *a* 27 19 if acute, but if obtuse, it is the Complement of that to 180 Degrees, viz. 152 : 41.

CASE *the Second.*

Given two Sides and an Angle oppoſite, to find the contain'd Angle.

Given { *a b* 42 12 / *b c* 35 20 / *c* — 32 12 } Required *b*. *Plate* 4. *Fig.* 18.

Let fall the Perpendicular *b d* by Solution the 9th; and then becauſe the Angle *a b c* is required, find the Angle *d b c* by the catholick Proportion, where you find the Side *b c*, is the middle Part, and the other extream Conjuncts, therefore the Proportion inverted is,

As Tang. Comp. of c —— 32 12 — 10.20084
To Radius —— —— — 90.00 — 10 00000
So Sine Comp. of $b\,c$ —— 35.20 — 9.91158

To Tang. Comp. of $c\,b\,d$ — 62.48 — 9.71074

The Angle $c\,b\,d$ being thus found, find the Angle $a\,b\,d$, by Concluſion the eighth, thus;

As Tang. of $a\,b$ —— — 42.12 Co ar 0.04252
To Sine Comp. of $c\,b\,d$ — 62.48 — — 9.66001
To Tang. of $b\,c$ —— 35.20 — — 9.85059

To Sine Comp. of $a\,b\,d$—69. 4 — — 9.55312

The Angle $a\,b\,d$ —— —— 69— 4
Added to the Angle $c\,b\,d$ — 62—48

The Sum is the Angle $a\,b\,c$ 131—52

Case *the Third.*

Given two Sides, and an Angle oppoſite, to find the third Side.

Given $\left\{\begin{matrix} a\,b\ 42\ 12 \\ b\,c\ 35\ 20 \\ c\text{—}32\ 12 \end{matrix}\right\}$ Required $a\,c$. *Plate* 4. *Fig.* 19.

Let fall the Perpendicular $b\,d$, by Solution the ninth; and then becauſe $a\,c$ is required, find $d\,c$ thus;

As

As Tangent-complement of one Extream-conj. *b c*: } 35.20.10.14941
To Radius —— —— —— —— 90 00.10.00000
So Sine-comp. of Middle-part 32.12. 9.92747

To Tang. of the other Extr. Conj. *d c* } 30.57. 9.77806

Then by Concluſion the ſixth.

As Sine-comp. of *b c* — 35.20 Co. ar. 0.08842
To Sine-comp. of *d c* — 30 57 — — 9.93329
So Sine-comp. of *a b* — 42.12 — — 9.86970
To Sine-comp. of *a d* — 38.51 — — 9 89141

The Segment *a d* —— —— 38.51
Added to the Segment *d c* — 30.57

The Sum is the whole Baſe *a c* 69.48

The ſecond and third Caſes may be ſolved without the Concluſions: As for Inſtance, In the ſecond Caſe where the contain'd Angle is required, having two Angles, and their Oppoſite Sides given or found, the third Angle may be found thus;

As Sine of half the Difference of the Sides
To Sine of half the Sum of the Sides,
So Tang. of half the Difference of the Angles
To Tang. Comp. of half the Angle required.

And in Caſe the Third, the third Side is thus found;

As Sine of half the Difference of the given Angles
To Sine of half the Sum of the given Angles,
So Tangent of half the Difference of the Sides
To Tangent of half the Side required.

Case *the Fourth.*

Two Angles, and a Side oppoſite to one of them given, to find the Side oppoſite to the other, if it be known whether the Side ſought be more or leſs than a Quadrant.

Plate 4. *Fig.* 20. Given { *a b* — 42.12 / *a b c* — 131.52 / *a c b* — 32.12 } Required *a c*

By Axiom the ſecond.

As Sine of *c*	——	32.12	Co ar 0.27338
To Sine of *a b*	——	42.12	— 9.82719
To Sine of *b*	——	131.52	— 9.87198
To Sine of *a c*	— —	69.50	— 9 97255

NOTE, When a Side, or an Angle ſought, is known to be leſs than a Quadrant, the Sine, or Tangent found, is the Anſwer to the Queſtion;

Queſtion; but if more, its Complement to 180 Degrees is the Anſwer.

NOTE alſo, When you are to find a Sine, or Tangent, of a Degree above 90, it is found by ſubſtracting the given Degree and Minute, from 180 Degrees; the Remainder is the Sine of the Degree ſought: Thus the Sine of 48.8 is the Sine of 131.52, *&c.*

CASE *the Fifth.*

Given two Angles, and a Side oppoſite to one of them, to find the Side included.

Given { *a* — 27 19, *c* — 32 12, *a b* 40 12 } Required *a c* *Plate* 4. *Fig.* 2.

Let fall the Perpendicular *b d*, by Solution the ninth; then becauſe *a c* is required, find the Segment *a d*.

As Tang. Comp. of *a b* ——	42.12	10.04251
To Radius —— ——	90.00	10 00000
So Sine Comp. of *a* —— ——	27 19	9.94865
To Tangent of *a d* —— ——	38.51	9.90614

Then by Conclusion the seventh.

As Tang. of *c*	32.12	Co ar 0.20084
To Sine of *a d*	38.51	9.79746
So Tang. of *a*	27.19	9.71307
To Sine of *d e*	30.57	9.71137

Then to the Segment *a c* — 38 51
Add the Segment *d c* — 30 57

The Sum is the Side *a c* — 69.48

CASE *the Sixth.*

Given two Angles, and a Side oppofite, to find the third Angle.

Plate 4. *Fig.* 22. Given { *c* — 32 12, *a* — 27 19, *b c* — 35.20 } Required *b*

Let fall the Perpendicular *d b*, by Concluſion the ninth; and then becauſe the Angle *a b c* is required, find the Angle *d b c*.

As Tang. Comp. of *c*	32 12	10.20084
To Radius	90 00	10.00000
So Sine Comp. of *b c*	35.20	9.91158
To Tang. Comp. of *c b d*	62 48	9.71074

Then

Then by Conclusion the fifth.

As Sine Comp. of c —— 32 12 Co ar 0.07254
To Sine of $c\ b\ d$ —— 62 48 —— 9 94910
So Sine Comp of a —— 27 19 —— 9.94865

To Sine of $a\ b\ d$ —— 69. 3 —— 9.97029

CASE *the Seventh.*

Given two Sides, and an Angle included, to find either Angle.

Given $\left\{\begin{matrix} a\ b & 42\ 12 \\ b\ c & 35\ 20 \\ b & 131\ 52 \end{matrix}\right\}$ Required a *plate 4. fig. 23.*

In this Case the Perpendicular may fall either from a or c, by Solution the ninth; but were there is that Variety, observe to let fall the Perpendicular from the End of a given Side, and oppofite to a given Angle, and also oppofite to the Angle required: As in this Case, or next the Side required, as in Case the ninth.

Then in the Right-angled Triangle $b\ d\ c$, Right-angled at d, there is given the Angle $d\ b\ c$ (because it is the Complement of the Angle $a\ b\ c$, to 180) and the Hipothenuse $b\ c$, to find the Base $b\ d$ thus;

As Tang. Comp. of *b c*	35.20	10.14941
To Radius	90.00	10.00000
So Sine Comp. of *d b c*	48. 8	9.82439
To Tang. of *d b*	25.19	9.67498

Then by Concluſion the ſeventh.

As the Sine of the Whole-baſe *a d*	67.31	0.03433
To the Tang. of *d b c*	48. 8	10.04759
So Sine of the Segment *b d*	25.19	9.63106
To Tang. of *a*	27.19	9.71298

CASE *the Eighth.*

Given two Sides, and an Angle included to find the third Side.

Plate 4. *Fig.* 24. Given { *a b* - 42 12, *b c* - 35 20, *b* — 131 52 } Required *a c*

Here by Solution the ninth, the Perpendicular may fall either from *a*, upon *c b* continued, or from *c*, upon *a b* continued to *d*, as here: And then as in Caſe the ſeventh, there is by Conſequence given the Angle *c b d*, by which, and the Hipothenuſe *c b*, you find the Baſe *d b*: As in Caſe the ſeventh, which there, is found to be 25 19

Then

Then by Concluſion the fifth.

As Sine Comp. of $d\,b$ —— 25.19 Co ar 0.04386
To Sine Comp. of $b\,c$ —— 35.20 —— 9.91158
So Sine Comp. of $a\,d$ —— 67.31 —— 9.58253

To Sine Comp. of $a\,c$ —— 69.49 —— 9.53797

CASE *the Ninth.*

Given two Angles, and a Side included to find either Side.

Given $\left\{\begin{matrix} a - 27.19 \\ b - 131.52 \\ a\,b - 42.12 \end{matrix}\right\}$ Required $b\,c$ *Plate 4. Fig. 25.*

Here by Solution the ninth, the Perpendicular may fall from a or b; but in this Caſe let it fall from b, becauſe it is the Concourſe of the Sides given and required: Then to find the Vertical Angle $a\,b\,d$, where $a\,b$ is Middle Part, and a and b Extreams Conjunct; therefore,

As Tang. Comp. of a —— 27.19 — 10.28692
To Radius —— —— 90.00 — 10.00000
So Sine Comp. of $a\,b$ —— 42.12 — 9.86970

To Tang. Comp. of $a\,b\,d$ — 69. 4 — 9.58278

From

From the whole Angle $a\,b\,c$ —— 131.52
Subſtract the Angle $a\,b\,d$ —— 69. 4

Remains the Angle $c\,b\,d$ —— 62.48

Then by Concluſion the eighth.

As Sine Comp. of $c\,b\,d$ — 62.48 Co ar 0.33999
To Sine Comp. of $a\,b\,d$ — 69. 4 — — 9.55301
So Tang of $a\,b$ —— —— 42.12 — — 9.95748

To Tang. of $b\,c$ —— 35.19 — — 9.85048

CASE *the Tenth.*

Given two Sides, and an Angle included to, find the other Angle.

Plate 4. *Fig.* 26. Given $\left\{\begin{matrix} a — 27\ 19 \\ b - 131\ 52 \\ a\,b - 42.12 \end{matrix}\right\}$ Required c

Here by Solution the ninth, and the General Rule in Caſe the ſeventh, the Perpendicular muſt fall from b, upon $a\,c$; then find the Vertical Angle $a\,b\,d$, by the Proportion in Caſe the ninth, which is there found to be 69 4.

Then

Then by Concluſion the fifth.

As Sine of *a b d* ——— 69. 4 Co ar 0.02966
To Sine Comp. of *a* ——— 27.19 ——— 9.94865
So Sine of *c b d* ——— 62.48 ——— 9.94910

To Sine Comp. of *c* ——— 32.13 ——— 9.92741

CASE *the Eleventh.*

Given three Sides, to find an Angle.

Given { *a b* 42 12 / *b c* 35 20 / *a c* 69 48 } Required *a* *Plate 4. Fig. 27.*

In this Caſe the Perpendicular muſt be let fall from the Angle *b*, upon the Side *a c*, by Concluſion the twelfth, and the Rules following.

Then by Concluſion the twelfth.

As Tang. of ½ the Baſe — 34.54 Co ar 0.15639
To Tang. of ½ the Sum of the Sides, } 38.46 ——— 9.90475
So Tang. of ½ Difference of the Sides, } 3.26 ——— 8.77811

To Tang. ½ Diff. of Seg. of the Baſe } 3.57 ——— 8.83925

To

To the Half-baſe ——— 34.54
Add the Half-difference ——— 3.57

The Sum is the greater Segment ——— 38.51
Or the Difference is the leſſer Segment - 30.57

— *Then by Axiom the firſt.*

As Radius ——— 90.00—10.00000
To Tang. of *a d* ——— 38.51— 9.90604
So Tang. Comp. of *a b* —— 42.12—10.04251

To Sine Comp. of *a* ——— 27.20— 9.94855

CASE *the Twelfth.*

Given three Angles, to find a Side.

Plate 4. Fig. 28. Given { *a* — 27 19 / *b* - 131 52 / *c* — 32 12 } Required *a b*

NOTE, Before you can reſolve this Caſe, you may turn the Angles into Sides, and the Sides into Angles; and then the Operation in the ſupplemental Triangle will be the ſame as in Caſe the eleventh; only in that Converſion, you muſt take the Complement of the greateſt Angle to 180, as in Solution the ſeventh.

The

The Given-angles being thus turned into Sides, and the Contrary; the Operation is the ſame as if you had

Given { *a b* — 27 19, *b c* — 32 12, *c a* — 48 8 } Required *a* *Plate 4. Fig. 27.*

By Concluſion the twelfth, let fall the Perpendicular upon the greateſt Side *a e*, from its oppoſite Angle *b* —— then;

As Tang. ½ the Baſe *a c* — 24. 4 Co ar 0.35006
To Tang. half Sum of the Sides *a b* and *b c* } 29.45 — — 9.75705
So Tang. half Differ. of the ſaid Sides } 2.26 — — 8.62834

To Tang. half Differ. Segm. of the Baſe } 3. 7 — — 8.73545

Half Baſe —— —— —— 24. 4
Half Differ. Segm. of the Baſe 3. 7
Their Sum is the greater Baſe *d c* 27.11
Their Differ. is the leſſer Baſe *a d* 20.57

Then by Axiom the firſt.

As Radius —— —— —— 90.00—10.00000
To Tang. Comp. of *a b* —— 27.19—10.28692
So Tang. of *a d* —— —— 20.57— 9.58304

To Sine Comp. of *a* —— — 42.10— 9.86998

The

The Angle *a* in this Triangle, being equal to the Side *a b*, viz. 42 10, is the Anſwer required.

The ſame Anſwer is produced by the Operation grounded upon Axiom the third, with only the Difference of a Minute or two, occaſioned by the omiting half Minutes in taking half of an odd Number, as appears by the following Calculation;

For the Side *a b*, the Operation is,

Angle *a*	27.19	Adj. Ang.	S Co. ar. 0.33828
Comp. *b*	48. 8		S Co. ar. 0.12802
Angle *c*	32.12		
Sum —	107:39	½ Sum Si. 53.21 —	9.90699
½ Sum -	53:49½	Remaind. 21 37½ —	9.56646
An. *c* ſub.	32:12		
		Sum of the 4 Log.	19.93975
Rem. —	21.37½	½ Sum Sine 21- 6 —	9.96987

21 6 doubled, is 42 12, the Side *a b* required

See a further Improvement of this Caſe, in the laſt Problem but one of Aſtronomy.

CHAP.

CHAP. IV.

ASTRONOMY.

SECT. I.

Aſtronomical Definitions.

NOTE, Where Letters are refer'd to, and no Figure mention'd, the ſaid Point, or Arch, *&c.* mark'd out by theſe Letters, may be found in all the 5 Projections; but where the Point, or Arch, cannot be ſeen in all, I ſhall refer to the Figure that beſt illuſtrates them: As Fig. 1. Fig. 2, *&c.* *NOTE*, All theſe Definitions refer to Plate 5.

POINTS.

POINTS.

The Zenith is the Point directly over our Heads, as *Z*; and the Nadir is the Point under our Feet, as *N*: Plate 5. Fig. 1.

The Poles of the Equinoctial, are the North-pole, as *P*, in all the Figures; and the South-pole, as *S*, Fig. 1. and 5. theſe are the Poles of the World —— The Pole of any great Circle is every Way 90 Degrees diſtant from it: Thus the Zenith and Nadir are the Poles of the Horizon, *&c.*

Great CIRCLES.

The Meridians are great Circles paſſing through the Poles of the World, and cutting the Equinoctial at Right-angles: As the Primitive-circle, Fig. 1, and the Right-circle *n z s*, Fig. 2 —— Of the Meridians, Two are called Colures, *viz.* The Solſtitial-colure, as the Primitive-circle, Fig. 1. (which cuts the Ecliptick in Cancer and Capricorn) And the Equinoctial-colures, which cuts the Ecliptick in Aries and Libra: As the Right-circle *P L S*, Fig. 1, and *E P W*, Fig. 3.

The Equinoctial is a great Circle that divides the Northern and Southern Hemiſphere, and cuts all Meridians at Right-angles: As *E L Q*, Fig. 1. *W Q E*, Fig. 2. The Primitive-circle Fig. 3, *E Q W*, Fig. 4, and *H Q O*, Fig. 5.

The

The Eclipſick makes an Angle of 23 30, with the Equinoctial in the firſt Point of Aries and Libra: As *E L C*, Fig. 1 — *W* ⊙ *E*, Fig. 2. and 3 — The Primitive, Fig. 4, and *H C* ⊙ *O*, Fig. 5. In it the twelve Signs are reckoned from Weſt to Eaſt: As ♈ Aries, ♉ Taurus, ♊ Gemini, ♋ Cancer, ♌ Leo, ♍ Virgo, ♎ Libra, ♏ Scorpio, ♐ Sagittarius, ♑ Capricorn, ♒ Aquarius, ♓ Piſces; each Sign containing 30 Degrees, makes in all 360 Degrees, as in all other great Circles.

The Horizon divides that part of the Sphere that is above the Earth, from that that is under the Earth: as *H L O*, Fig. 1 — The Primitive, Fig. 2, and *H a O*, Fig. 5.

The Azimuths are Circles interſecting each other in the Zenith and Nadir, and cuts the Horizon at Right-angles: As *Z* ⊙ *N*, Fig. 1 — ⊙ *Z R*, Fig. 4. And the Azimuth that cuts the Horizon in the Eaſt and Weſt Points thereof, is called the Prime-vertical: As *Z L N*, Fig. 1, and *W Z E*, Fig. 2. — And the Primitive, Fig. 5.

Circles of Longitude, cut the Ecliptick at Right-angles, and interſect each other in the Poles thereof: As *E A W*, and *C A K*, Fig. 4.

Leſſer CIRCLES.

They are commonly Parallel to ſome great Circle, and hence called Parallels: As Parallels of Altitude, as *s* ⊙ *U*: and Parallels of Declination, as *g* ⊙ *m*, &c. Fig 1.

The

NOTE, Our *Engliſh* Geographers and Navigators of Late, do commonly begin their Longitude at *London*, and Number Eaſt or Weſt from thence.

ANGLES.

The Hour of the Day is an Angle, made at the Pole, between the Meridian of the Sun, and the Meridian of the Place: Thus the Angle *Z P* ⊙, is the Hour from Noon, when the Sun is any where in the Meridian *P* ⊙ *S*, Fig. 1.

The Sun's Azimuth is an Angle at the Zenith, contain'd between the Azimuth that the Sun is in, and the Prime-vertical (if reckon'd from the Eaſt or Weſt) or the Meridian, if reckon'd from the North or South: Thus the Angle ⊙ *Z L*, is the Sun's Azimuth from the Eaſt or Weſt; but ⊙ *Z E*, is the Sun's Azimuth from the South, Fig. 1.

The Sun's greateſt Declination is the Angle that the Ecliptick makes with the Equator: As the Angle *b L* ⊙, Fig. 1.

NOTE, All theſe and other Angles in Aſtronomy, may alſo be reckon'd Arches of thoſe great Circles that are 90 Degrees diſtant from the Angular-point: Thus the Hour is an Arch of the Equinoctial: The Azimuth is an Arch of the Horizon, *&c.* Of which ſee more in the foregoing Definitions, and in Prob. 9. of Spherical Geometry.

SECT.

SECT. II.

Queſtions in ASTRONOMY.

ASTronomy is no more but the Application of Spherical-trigonometry, or ſuppoſing the Arches of Circles, which conſtitute your given Triangle, to be Arches of the Horizon, Equator, Ecliptick, *&c.* and thus by truly knowing what Circles the Longitude, Declination, Right-aſcenſion, *&c.* are Arches of, as is already taught at large in Chap. 4, Sect. 1, You have nothing to do in your Aſtronomical-operations, but what you are already taught in the Operations in Spherical-trigonometry; only to make the Work more intelligible, I ſhall inſtance in ſome of the moſt uſeful Queſtions in Aſtronomy; and for Variety, I ſhall perform the Operations by my Lord *Nepair*'s Proportion, that the Learner may underſtand the Variety of Proportions, whereby Spherical-trigonometry and Aſtronomy may be performed.

NOTE, Whereas in moſt Caſes in Aſtronomy, when you project them, you firſt draw the Primitive-circle, and croſs it with two Diameters (as Plate 5, Fig. 1, the Primitive-circle *Z H N O*, is croſt with two Diameters *Z N* and *H O*) therefore to avoid needleſs Repetitions, when you ſee at the firſt, *P C Q*, it ſignifies, Draw the Primitive-circle, and Quarter it, *viz.* Croſs

it with two Diameters, as above deſcribed, and then proceed as directed.

PROBLEM I.

Given { Sun's Longitude 60: 0 / Greateſt Declin. 23:30 }

Required Sun's preſent Declination.

Plate 5. *Geometrical Projection.*

I ſhall Inſtance in this upon all the five Plains; and firſt, Upon the Plain of
Fig 1. the Meridian.

NOTE, Although the Solution of this Problem has no regard to a given Latitude; yet becauſe it ſeems more natural to ſuppoſe a given Latitude, I have inſtanced in Latitude 54 40 North.

P C Q, and from *O* ſet off the Lat. 54 40 to *P*, and likewiſe from *H* to *S*, and draw *P S*, the Axis of the World; and at Right-angles it, draw *Æ Q* the Equator; and at an Angle of 23 30 to that, draw *E C* the Ecliptick, upon which (becauſe Longitude is an Arch of the Ecliptick) ſet off ☉'s Longitude 60, from *L* to ☉, and draw the Meridian *P*, ☉, *S*; then (becauſe the Declination is an Arch of the Meridian) the Arch *h*, ☉ meaſured by Prob. 8. Var. 3. is the Declination required.

2. Un-

2. Upon the Plain of the Horizon. *Fig.* 2.

P C Q, then is the Center *Z* the Zenith, and *P* the Pole; then 90 Degrees from *P*, draw the Equinoctial *W Q E*, and 23 30, from that the Ecliptick *W C E*; ſet off 60 Degrees the ☉'s Longitude from *E* to ☉, and draw the Meridian *P* ☉ *b R*, the Arch ☉ *b* is the Declination required.

3. Upon the Plain of the Equator. *Fig.* 3.

P C Q, then is the Primitive Circle the Equator, and *P* the Pole; draw *E C W* the Ecliptick, to make an Angle of 23 30, with the Primitive at *E* and *w*, and ſet off ☉'s Longitude 60, ♋ from *E* to ☉, and draw the Meridian *b* ☉ *P*; then by ☉ meaſured by Prob. 8. Var. 2. is the ☉'s Declination required.

4. Upon the Plain of the Ecliptick *Fig.* 4.

P C Q, then 23 30 from the Ecliptick, as through *Q*, draw the Equator *E Q W* continued round; becauſe the Triangle falls without the Primitive-circle; and having found *P* the Pole of the World, which is known to be 23 30 diſtant from *A* the Pole of the Equator, draw the Meridian *R P* ☉ *b*; the Arch ☉ *b* is the Declination required.

NOTE, Any Arch without the Primitive-circle, is meaſured by the ſame Rules in Prob. 8. that an Arch within the Primitive-circle is

meaſured; and here I have contrived the Projected-triangle to fall without, to ſhew the Reader the Variety.

5. Upon the Plain of the Prime-vertical *Fig.* 5.

P C Q, then is *Z* the Zenith, draw *HC O* the Ecliptick, and *H Q O* the Equator; ſet off ☉'s Longitude 60, 0 from *O* to ☉, and draw the Meridian *P*, ☉, *S*, to cut the Equator in *b*; the Arch *b* ☉ is the Declination required.

NOTE, The Triangle *o b* ☉ falling within the Primitive, all that Part falling without, expreſs'd by prick'd Lines, might be omited; only here I have inſerted it to let the Learner ſee the Reaſon of his Work.

Trigonometrical Calculations.

The Triangle *L b* ☉, Fig. 1, or *E b* ☉, Fig. 2, 3, and *w b* ☉, Fig. 4, or *O b* ☉, Fig. 5, is the Triangle here propoſed, in which you have given *L* ☉, Fig. 1, or *E* ☉, Fig. 2, 3, or *w* ☉, Fig. 4, or *O* ☉, Fig. 5, the Sun's Longitude 60 0; and the Angle *L*, Fig. 1, or *E*, Fig. 2 and 3, *w*, Fig. 4, or *O*, Fig. 5, to find the Arch *b* ☉; by Caſe the ſecond.

Here the Sun's Declination required is Middle-part, and the other two, Extreams-disjunct; therefore,

As

As Radius ——— 90: 0:10.00000
To Sine of Greateſt-declination 23:30 9:60070
So Sine of Sun's Longitude —— 60:00. 9.93753

To Sine of Preſent-declin. req. 20 12 9.53823

PROBLEM II.

Given { ☉'s Longitude 60 0 / ☉ Greateſt-de. 23 30 } Req. ☉ Rig. Aſc.

'Tis projected as Prob. 1. being the ſame Data.

The Given angle is Middle-part, and the other Extreams-conjunct — Therefore by Caſe the firſt.

As Tang. Comp. ☉ Long. —— 60 0. 9.76144
To Radius ——— 90 0 10.00000
So Sine-com. Greateſt-dec. S.C. 23 30. 9.96240

To Tang. of ☉ Right-aſcenſ. 57:48:10,20096

PROBLEM. III.

Given { ☉ Greateſt-declin. / Preſent-declinati. } Req. ☉'s Long.

Greateſt-declination 23 30 Preſent-declination 20 12 *Plate 5. Fig. 1.*

To project it upon the Plain of the Meridian.

P C Q, and draw *p S*; alſo draw *E Q*, the Equator, and *E C*, the Ecliptick, *&c.* as in Prob. 1. of Aſtronomy; then draw the Parallel of Declination g ☉ *m*, and where it cuts the Ecliptick, as in the Point ☉, draw the Meridian *P* ☉ *S*; then is *L* ☉ the Longitude required by Caſe the twelfth.

As Sine-ext. Diſj. greateſt Dec. 23.30. 9.60070
To Radius ——— ——— ——— 90 00.10.00000
So Sine Mid. part preſent Decl. 20 12. 9.53819

To Sine-ext. Diſj. ☉ Longit. — 60 0 9.93749

PROBLEM IV.

{Given ☉'s Declination — 20 12
{Greateſt Declination —— 23.30

Plate 5. *Fig.* 1. Required ☉ Right-aſcenſion

Projected as Prob. 3. —— Then by Caſe the tenth

As Radius ——— ——— ——— 90 0.10.00000
To Tangent-compl. Extream-conj. greateſt Declin. T C } 23 30.10.36170
So Tang. Ext. Con. preſent Dec. 20 12. 9.56576

To Sine Mid. part Right-aſcen. 57:48 9.92746

PRO-

PROBLEM V.

Plate 6. Fig. 1.

Given { ☉ greateſt Dec. 23.30 / Right-aſcenſion 57.48 } Req. ☉'s Lon.

P C Q, and as you have no reſpect to Latitude in this, as well as all the foregoing Problems, you may let your Perpendicular Diameter repreſent the Axis of the World, and the other the Equator; then at an Angle of 23 30, with the Equator, draw *E C* the Ecliptick; then upon the Equator, ſet off the Right-aſcenſion (becauſe the Right-aſcenſion is always an Arch of the Equator) from *L* to *b*, and draw the Meridian *P b S*, and obſerve where it cuts the Ecliptick: As in the Point ☉, the Arch *L* ☉, is the ☉'s Longitude required. Then by Caſe the ninth.

As Tang. Ex. Conj. Right-aſc. 57:48 — 10.20084
To Radius — — — — — — 90:00.10.00000
So Sine Comp. Mid. pt. gr. Dec. 23:30. — 9.96240

To Tan. Com. Ex. Conj. Long. 60: 0 — 9.76156

PROBLEM VI.

Given { ☉'s greateſt Dec. 23.30 / Right-aſcenſion - 57.48 } Req. ☉ Decl.

Projected as Prob. 5 —— Then by Caſe the ſeventh

As Tan. Com. Ex. Con. Gr. De. 23.30 10.36170
To Radius ——— ——— — 90 00.10.00000
So Sine Mid. Pt. Right-aſcen. 57.48. 9.92747

To Tang. Ex. Conj. ⊙ Declin. 20:12 9.56577

PROBLEM VII.

Plate 6. Fig. 1. Given { Lat. of the Place 54 40 / ⊙ Declination - 20 12 } Required.

Aſcenſional difference ——— ——— Prob. 7
⊙ Riſe and Set ——— ——— —— Prob. 8
Length of Day or Night ——— —— Prob. 9
⊙'s Amplitude ——— ——— —— Prob. 10
What Hour ⊙ is Eaſt ——— —— Prob. 11
⊙'s Altitude when Eaſt ——— Prob. 12
⊙ Altitude at 6 ——— ——— —— Prob. 13
⊙'s Azimuth at 6 ——— ——— — Prob. 14

P C Q, ſet off the Latitude 54 40, from *O* to *P*, and from *H* to *S*, and draw the Axis *P*, *S*, and at Right-angles to it, draw the Equator, *Æ Q*, and at an Angle of 23 30, draw *E C* the Ecliptick; draw the Parallel of Declination *G* ⊙ *M*, diſtant from the Equator 20 12, equal to the Given-declination; and where it cuts the Ecliptick, as in ⊙, draw the Meridian *P* ⊙ *S*, and the Azimuth *Z* ⊙ *N*; alſo where the Parallel cuts the Horizon, as in *D*, draw the Meridian *P D S*; and where the Parallel cuts the Prime-vertical, as in 3, draw the

the Meridian *P 3 S*; alſo where the Parallel cuts the Axis *P S*, as in 5, draw the Azimuth *Z 5 N*; which done, the Arch *L y* is the Aſcenſional-difference, *L D* the Amplitude from the Eaſt or Weſt, becauſe *G M* is the Parallel the Sun moves in that Day, and the Point *D* is the Point of Sun's Riſing: *D O* the Amplitude from the North, the Angle 3 *P L* equal to the Arch 2 *L*, is the Hour from 6 that the Sun is Eaſt or Weſt, *viz.* In the Point 3, the Point 3 being the Point that the Sun is in when Eaſt or Weſt; and hence the Arch 3 *L* is the Sun's Altitude when Eaſt or Weſt, and the Arch 5 6, is the Sun's Altitude at 6, and the Arch *L* 6, is the Sun's Azimuth at 6, taken from the Eaſt or Weſt, *&c.* Then for Trigonometrical-operation.

Firſt, For the Aſcenſional-difference.

In the Triangle *L y D*, you have given the Angle at *L* the Comp. Latitude, and the Leg *D y* the Declination to find *L y* the Aſcenſional Difference: Therefore by Caſe the Tenth.

As Radius	90	0.10.00000
To Tang. of Latitude	54 40.	10.14941
So Tang. of Declination	20 12.	9.56576
To Sine of Aſcenſional-differ.	31:16.	9.71517

PROBLEM VIII.

To find the Time of Sun's Rising and Setting.

The Ascensional Difference 31 16 converted into Time, allowing 15 Degrees to one Hour, and one Degree to 4 Minutes of Time, gives the Hours and Minutes that the Sun Rises before 6 in Summer, or after 6 in Winter: Thus 31 16, gives 2 Hours 5 Minutes, which subtracted from 6, leave 3 Hours 55 Minutes in the Morning for Sun's Rising; or added to 6, makes 8 Hours 5 Minutes at Night for Sun's Setting, *&c.* and the contrary in South Declination, when the Sun Rises after, and Sets before 6; and then, instead of adding, you must substract, and the contrary ——

PROBLEM IX.

To find the Length of the Day or Night.

The Sun's Rising found by Prob. 8. being doubled, gives the Length of the Night, and double Sun's Setting, gives the Length of the Day: And thus, as by Prob. 8. you find Sun Sets at 8 Hours 5 Minutes, which doubled, gives 16 Hours 10 Minutes for the Length of the Day; and the Sun's Rising, *viz.* 3 Hours 55 Minutes doubled, gives the Length of the Night,

Night, which ſo found, is 7 Hours 50 Minutes, *&c.* ——

PROBLEM X.

Plate 6. Fig. 1.

Latitude and Declination given, to find the Amplitude.

In the former Projection from the ſame Data, you have given in the Triangle *L y D*, the Angle at *L* the Complement of Latitude, and the Leg *D y* the Declination, to find the Hypothenuſe *L D* the Amplitude; and here the Declination *D y* is Middle Part, and the other Extream Disjuncts; therefore by Caſe 12.

As Sine Comp. Latitude, —— S C 54 40		9.76218
To Radius; —— —— ——	90 00	10.00000
So Sine of Declination, ——	20 12	9.53819
To Sine of Amplitude —— ——	36 46	9.77601

This is the ſame Proportion that ariſes from the Globical Projection.

PROBLEM XI.

Latitude of the Place 54 40, and Sun's Declination 20:12, given, to find what Hour and and Minute the Sun will be due Eaſt or Weſt.

Projected as in Prob. 7, being the ſame Data.

This falls in the Triangle *L* 2 3, in which you have given the Angle 2 *L* 3, the Latitude 54 40, and the Leg 2 3, the Declination to find the Leg 2 *L*, the Hour from 6 when the Sun is in the Point 3, *viz.* Due Eaſt or Weſt, and falls in Caſe the Tenth of Right-angled Spherical Triangles; therefore,

As Radius, —— —— ——	90	0:10.00000
To Tangent Compl. Latitude,	54 40.	9.85059
So Tangent Declination, ——	20.12.	9.56576
To Sine of the Hour from 6,	15. 7	9.41635.

Which gives one Hour to be added to 6 in the Morning, or ſubſtracted from 6 in the Afternoon, the Sum 7 is the Time when the Sun is Eaſt, and the Remainder 5 the Time when he is due Weſt, *&c.* ——

Plate 6. *Fig.* 1. PROBLEM XII.

Having the Latitude 54 40, and Declination 20 12, to find the Sun's Altitude when Eaſt or Weſt.

This falls alſo in the Triangle *L* 2 3, the Angle *L* and Leg 2 3, given to find the Hipothenuſe *L* 3, by Caſe 12.

As

As Sine of Latitude,	54.40	9.91158
To Radius;	90:00.	10.00000
So Sine Declination;	20:12.	9.53819.
To Si. of ⊙ Alt. when E. or W.	25: 3:	9.62661.

PROBLEM XIII.

Plate 6. Fig. 1.

By the Latitude 54.40, and Declination 20 12, to find the Sun's Altitude at 6.

This projected as in Prob. 7, falls in the Triangle *L* 5 6, in which is given the Angle *L*, the Latitude, 54 40, and *L* 5, the Declination, 20 12, to find the Side 5 6, the Point 5, being the Point the Sun is in at 6, and the Arch 5 6 his Altitude above the Horizon at that Time; therefore by Caſe 2.

As Radius,	90	0.10.00000
To Sine Latitude;	54:40.	9.91158
So Sine Declination,	20 12.	9.53819.
To Sine of Altitude at 6.	16:21	9.44977.

PROBLEM XIV.

Plate 6. Fig. 1.

Given the Latitude 54 40, and Declination 20 12, to find Sun's Azimuth at 6, which falls in the ſame Triangle *L* 6 5, in which is given, as in Prob. 13, to find the Leg *L* 6, by Caſe the Firſt.

As

As Tang. Comp. Decl. — T C 20.12.10.43424
To Radius; — — — — 90.00.10.60000
So Sine Comp. Latitude — S C 54 40. 9.76218

To Tang. Sun's Azimuth at 6. 12. 1 9.32794

Plate 6. Fig. 1.

PROBLEM XV.

Latitude and Aſcenſional Difference given, to find Sun's Declination.

This, and the following Problems, are only other Parts given and required in the fore-mentioned Triangles, as the Learner may eaſily ſee by what is already laid down in the fore-going Problems; and therefore I ſhall not need to inlarge, but ſhall lay down the Queſtions, and leave the Operations for the Reader's Practice.

In the Triangle *L y d*, there is given *L y*, the Aſcenſional Difference 31 16, and the Angle *L* the Complement of Latitude 35 20, to find *D y*, the Declination, by Caſe the Seventh.

As Tang. Comp. of *L*,
To Radius;
So Sine of *L y*,
To Tang. of *D y*.

PRO-

PROBLEM XVI. *Plate 6. Fig. 1.*

Given the Latitude of the Place 54 40, and the Aſcenſional Difference 31 16, to find the Sun's Amplitude.

In the Triangle *L y D*, there is given the Angle at *L*, and the Leg adjacent *L y*, to find the Hypothenuſe *L D*, by Caſe the Ninth.

As Tangent *L y*,
To Radius;
So Sine of Complement of *L*,
To Tangent Complement of *L d*.

PROBLEM XVII. *plate 6. Fig. 1.*

Given the Latitude of the Place 54 40, and the Amplitude of the Sun 36 40, to find the Sun's Declination.

In the Triangle *L y D*, is given the Angle at *L* the Complement of Latitude, and *L D* the Amplitude to find the Declination *D y*, by Caſe the Second.

As Tangent Complement of *L*,
To Radius;
So Sine Complement of *D L*,
To Tangent of *D y*.

PRO-

Plate 6. Fig. 1. PROBLEM XVIII.

Given the Latitude 54 40, and Sun's Amplitude 36 40, to find the Aſcenſional Difference.

In the Triangle *L y d*, is given the Angle *L*, and the Hippthenuſe *L D*, to find *L y* by Caſe the Firſt.

As Tangent Complement of *L D*,
To Radius;
So Sine Complement of *L*,
To Tangent *L y*.

Plate 6. Fig. 1. PROBLEM XIX.

The Sun's Declination 20 12, and Aſcenſional Difference 31 16, given to find the Latitude of the Place.

In the Triangle *L y d*, is given *L y*, and *D y*, to find the Angle *L*, by Caſe the Thirteenth.

As Tangent *D y*,
To Radius;
So Sine of *L y*,
To Tangent Complement of *L*.

PRO-

PROBLEM XX.

Plate 6. Fig. 1.

Given as in Prob. 19, to find Sun's Amplitude *L d*, by Caſe the Fourteenth.

As Radius,
To Sine Complement *L y*;
So Sine Complement *D y*,
To Sine Complement *L D*.

I might advance to a great Number of Problems, which may be deduced from the ſeveral Right-angled Triangles, contain'd in this Figure; but not willing needleſly to encreaſe the Number, I ſhall confine my ſelf only to thoſe where the Latitude is required, they being of moſt Uſe in Practice at Sea, the Artiſt being commonly fitted with Books, Inſtruments, *&c.* whereby the Sun's Altitude Declination, *&c.* is attain'd without Trigonometrical Calculation; and by the Help of theſe Problems following, the Latitude may ſometimes be attain'd in a Morning, or Evening, when an Obſervation cannot be had at Noon.

Plate 6. PROBLEM XXI.
Fig. 1.

Given the Sun's Declination 20 12 North, and the Amplitude 36 40, to find the Latitude of the Place.

In the Triangle *L y D*, is given the Hipothenuſe *L D*, and the Leg *L y*, to find the Angle *L*, by Caſe the Fifth.

As Sine *L D*,
To Radius;
So Sine of *D y*,
To Sine of *L*.

Plate 6. PROBLEM XXII.
Fig. 2.

The Sun's Amplitude 36 40, and Aſcenſional Difference 31 16, given, to find the Latitude of the Place.

In the Triangle *L y D*, you have given the Hipothenuſe *L D*, and the Leg *L y*, to find the Angle *L*, by Caſe the Fourth.

As

As Radius,
To Tangent *L y*;
So Tangent Complement of *L D*,
To Sine Complement of *L*.

PROBLEM XXIII. *Plate 6. Fig. 1.*

Given the Sun's Declination 20 12, and his Altitude at Six a Clock 16 22, to find the Latitude.

In the Triangle *L* 5 6, is given the Hipothenuſe *L* 5 the Declination, and the Leg 5 6 the Altitude at 6, to find the Angle *L* the Latitude, by Caſe the Fifth.

As Sine of *L* 5,
To Radius;
So Sine of 5 6,
To Sine of the Angle at *L*.

PROBLEM XXIV. *Plate 6. Fig. 1.*

Given the Sun's Declination 20 12, his Azimuth at 6 12 1, to find the Latitude.

In the Triangle *L* 5 6, is given *L* 5 the Declination, and *L* 6 the Azimuth at 6, to find the Angle at *L*, by Caſe the Fourth.

As

As Radius,
To Tangent *L* 6;
So Tangent Complement of *L* 5;
To Sine Complement of *L* required.

Or in the Quadrantal Triangle *L Z* 5, is given *L* 5 the Sun's Declination 20 12, and the Angle *Z* the Sun's Azimuth at 6, *viz.* 12 1, to find the Angle at *L*, the Complement of Latitude required, which by the Rules given Chap 3, for Quadrantal Triangles, falls under the following Proportion.

As Radius,		10.00000
To Tang. of the Ang. *Z* 5	12. 1	9.32809
So Tang. Com. Dec. *L* 5	20 12	20.43423
To Sine Com. Lat. *Z L* 5	54 39	9.76232

Plate 6. Fig. 1. **PROBLEM XXV.**

Given the Sun's Declination, together with his Altitude, taken due East or West, to find the Latitude.

In the Triangle *L* 5 6, is given the Declination *L* 5, 20 12, and the Sun's Altitude, when East or West, 25 3, the Leg 5 6, to find the Angle *L*, the Complement of Latitude, by Case the Fifth.

As

As Sine of *L* 5 the Declination,
To Radius;
So Sine of 5 6, the Sun's Altitude, when Eaſt or Weſt,
To the Sine Complement of *L* the Latitude required.

Or in the Quadrantal Triangle, *Z L* 5, is given *Z* 5, the Complement of Altitude, when Eaſt or Weſt; and *L* 5, the Declination, to find the Angle *L*, the Complement of Latitude.

As Sine of *L* 5 the Declination,
To Radius;
So the Sine Complement of *Z* 5 (which is the Sine of 5 6 the Sun's Altitude,)
To the Sine Complement of *L* (which is the Sine of Latitude) required.

SECT.

SECT. III.

Aſtronomical PROBLEMS *whoſe Solutions depend upon Oblique Spherical Triangles.*

Plate 6. Fig. 2.

PROBLEM I.

GIVEN the Sun's Altitude Declination, and Azimuth, to find the Hour of the Day.

EXAMPLE.

The Sun's Altitude, — 47 48
The Sun's Dec. North, 20 12 } Req. the Hour.
Azimuth from the Sou. 48 8

I have given this the firſt Place, becauſe it falls under the firſt Caſe of Oblique Spherical Triangles; my Deſign being (for the Learner's better Inſtruction) to inſtance one Queſtion in each Caſe, and to project each upon the Plain of ſuch a great Circle, that it may come under this,

General

General Rule for projecting Oblique Spherical Triangles.

When there is given two Sides, and an Angle make a given Side upon the Primitive, and the given Angle at the Primitive; but if two Angles and a Side make both the Angles at the Primitive, and let the given Side fall within, except it be the Side contain'd between the given Angles; but when only three Sides is given, as in Caſe 11, or three Angles, as in Caſe 12, you may project upon the Plain of any of the three Circles, whoſe Arches compoſe the Triangle.

NOTE, You may in many Caſes project the Triangle at the Center of the Primitive, as in Prob. 7 and 12 following.

It is moſt convenient to project this Problem upon the Plain of an Azimuth, becauſe the Sun's Altitude (which is an Arch of an Azimuth) is given, and the Sun's Azimuth is an Angle adjacent to that given Side; therefore, *P C Q*, and ſet the Sun's Altitude 47 48, upon the Primitive from *H* to ☉, and draw the Meridian of the Place *Z P n*, to make an Angle 139 52 with the Primitive (equal to the Sun's Azimuth from the North, or the Complement of the Azimuth from the South) by Prob. 2, Caſe 25 and becauſe the Sun is diſtant from the Pole 69 48, equal to the Complement of Declination, draw the Parallel *a P b*, at the

ſaid

ſaid Diſtance from the Sun, by Prob. 7, Caſe 3; and becauſe the Pole is ſomewhere in that Parallel, and alſo in the Meridian of the Place, it muſt be where they croſs each other, as in *P*; then through ☉ and *P*, draw the Meridian ☉ *P C*, to cut the Primitive in the oppoſite Points ☉ and C, and the Triangle is finiſhed; and in the Triangle *Z P* ☉, there is given *Z* ☉, the Complement of Altitude 42° 12', and ☉ *P*, the Complement of Declination 69° 48', and the Angle ☉ *Z P*, the Azimuth from the South, to find the Angle *Z P* ☉, the Hour from Noon, by Caſe the Firſt.

As Sine of ☉ *P*, —— 69 48 Co ar 0.02757
To Sine of ☉ *Z P*; —— 131 52 —— 9.87198
So Sine of ☉ *Z*, —— 42 12 —— 9.82718

To Sine of *Z P* ☉. —— 32 12 —— 9.72673

This 32° 12', reduced to Time, allowing 15 Degrees to an Hour, and every Degree 4 Minutes, gives 2 Hours 9 Minutes, *viz.* 15 Minutes after 9 in the Morning, or 9 Minutes paſt 2 in the Afternoon.

Plate 6. Fig. 3. PROBLEM II.

Given the Latitude of the Place, with the Sun's Altitude, and Hour of the Day, to find the Azimuth.

Latitude

Latitude North, ——— 44 40
Sun's Altitude, ——— 47 48 } to find the
Ho. from No. 2 Ho. 9 M. or 32 12 } Azimuth.

This muſt be projected upon the Plain of the Meridian, becauſe the Latitude (an Arch of the Meridian) and the Hour (an Angle adjacent) is given; therefore,

P C Q, ſet the Latitude 54 40, from *Q* to *P*, or its Complement from *Z* to *P*, draw *P* ⊙ *S*, to make the given Angle at *P* 32 12, by Prob. 3, Caſe 2, and draw the given Parallel of Altitude 47 48 above the Horizon, by Prob. 7, Caſe 2, as *b* ⊙ *c*; and where that cuts the Meridian *P* ⊙ *S*, as in the Point ⊙ draw the Azimuth *Z* ⊙ *N*, and the Triangle *Z P* ⊙ is finiſhed, in which you have given,

The Comp. Lat. *Z P*,
The Comp. Alt. *Z* ⊙, } To find the Azimuth,
The Angle *Z P* ⊙, — } ⊙ *Z P* by Caſe the Sec.

The Triangle *Z P* ⊙, muſt firſt be ſuppoſed to be divided into two Right-angled Triangles, by the Directions given in Caſe the Second, of Oblique Spherical Triangles; which done, the Operation is,

As Tang. Comp. of *Z P* ⊙, — 32 12 10.20084
To Radius; ——— ——— ——— 10.00000
So Sine Comp. of *Z P*, ——— 35.20 9.91158

To Tan. Com. of *P Z c*. ——— 62 48 9.71074

Then by Concluſion the Eighth.

As Tangent of ⊙ Z, —— 42:12 Co ar 0.04851
To Sine Comp. P Z e, — 62 48 — — 9.66002
So Tang. of Z P, —— 35 20 — — 9.85059

To Sine Comp. ⊙ Z e. — 69. 4 — — 9.55311

To the Angle P Z e, 62 48
Add the Ang. ⊙ Z e, 69 4

The Sum is P Z ⊙. 131.52. the Azimuth requi.

The Reaſon of this Concluſion, made uſe of in the laſt Operation of this Problem, and alſo of the reſt mentioned under Axiom the Second, of Spherical Trigonometry, may be deduced from theſe Conſiderations, *viz.*

Suppoſe in the Triangle Z P ⊙, beforementioned, the Perpendicular Z e, which divides it into two Right-angled Triangles, were given; and ⊙ Z, and Z P, which are now become Hipothenuſes, to the two Right-angle Triangles, were alſo given, to find the Vertical Angles, by Caſe the Fourth, the Proportion would be,

As Tangent Complement Z e,
To Radius;
So Tangent Complement Z P,
To Sine Complement P Z e.

As Tangent Complement Z e,
To Radius;
So Tangent Complement Z ⊙,
To Sine Complement ⊙ Z E.

The

The two firſt Terms in both Proportions being the ſame, take them both away, and the reſt will retain the ſame Proportion (*Euclid.* l. 5. p. 11.) viz.

As Tangent Complement $Z P$,
To Sine Complement $P Z E$;
So Tangent Complement $Z \odot$,
To Sine Complement $\odot Z e$.

And if the Tangent Complements are directly, the Tangents muſt be mutually, or reciprocally proportionable to the Sine Complements of their reſpective Angles; and by this Way of Reckoning, you may find the Ground of all the reſt of the Concluſions, always imagining the Perpendicular, which is common to both Triangles, to be given; and the other Parts given, to be the ſame in both Triangles, whether the Hipothenuſe, or Angle, at the Baſe, *&c.* And by this you may form two Proportions, whoſe two firſt Terms will be the ſame, which being deſtroyed, the other retaining the ſame Proportion, is the Concluſion required.

PROBLEM III.

Given the Latitude of the Place, with the Sun's Altitude, and the Hour of the Day, to find the Declination.

Latitude North, - 54 40
Sun's Altitude, — 47 48 } to find the Declinat.
Hour from Noon, 32 12 } ⊙ *P*, by Caſe the Sec.

Plate 6. Projected as Prob. 2.
Fig. 3.

Which being done, and the Perpendicular *Z e* let fall by Solution the Ninth; the Operation is,

As Tang. Comp. *Z P*, — —— 35 20 10.14948
To Radius; —— —— —— 10.00000
So Sine Comp. *Z P e*, —— — 32 12 9.92747

To Tangent *P e*. —— —— 30 57 9.77799

Then by Concluſion the Sixth.

As Sine Comp. *Z P*, —— 35 20 Co ar 0.08841
To Sine Comp. *e P*; —— 30 57 — — 9.93329
So Sine Comp. ⊙ *Z*, —— 42 12 — — 9.86970

To Sine Comp. ⊙ *e*. —— 38 51 — — 9.89140

To *P e*, —— 30 57
Add ⊙ *e*, — 38 51

Sum is ⊙ *P*, 69.48 the Complement of Declination, which taken from 90, leaves 20 12, the Declination required.

For

For other Methods of Solution, ſee Caſe the Third, of Oblique Spherical Triangles.

PROBLEM IV. *Plate 6. Fig. 3.*

Given the Sun's Altitude, Azimuth, and Hour from Noon, to find the Declination.

Altitude $H \odot$, — 47.48
Azimuth $\odot Z P$, 131 52 } to find the Dec. $\odot P$;
Hour $\odot P Z$, —— 32 12 } by Caſe the Fourth.

Projection upon the Plain of the Meridian $P C Q$, draw the Azimuth $Z \odot N$, to make with the Primitive the given Angle $\odot Z P$, 131:52; draw the Parallel $b \odot c$, diſtant from Z, equal to the Complement of Altitude 42 12; and where that cuts the Azimuth $Z \odot N$, as in $\odot$, is the Point the Sun is in; therefore through the Point $\odot$, by Prob. 4, of Spherical Geometry, draw the Meridian $P \odot S$, to make the given Angle 32 12, with the Primitive at P, and then the Triangle is finiſhed.

Now in the Triangle $Z P \odot$, is given $Z \odot$, the Complement of Altitude, $\odot Z P$ the Azimuth, and $Z P \odot$, the Hour to find $\odot P$, the Complement of Declination, by Caſe the Fourth, of Oblique Spherical Triangles, where the Caſe is propoſed in the ſame Numbers; and therefore I ſhall not make any Repe-

tition, but refer the Learner to that Caſe; the Complement of Declination is 69 50, and conſequently the Declination 20 10.

PROBLEM V.

Plate 6. Fig. 3.

Given the Sun's Altitude, Azimuth, and Hour of the Day, to find the Latitude.

Altitude, - 47 48
Azimuth, 131 52 } to find the Latitude by Caſe
Hour, — 32 12 } the Fifth.

The Data and Projection is the ſame, as in Prob. 4; but by Solution the Ninth, the Perpendicular falls without, as in Plate 6, Fig. 4; then in the Right-angled Triangle, $Z\ D\ \odot$, Right-angled at D, there is given the Hipothenuſe, $Z\ \odot$, 42 12, and the Angle, $\odot\ Z\ D$, (the Complement of $P\ Z\ \odot$, to 180) *viz.* 48 8, to find $Z\ D$.

As Tang. Comp. $Z\ \odot$, ———	42 12	10.04251
To Radius; ——— ——— ———		10.00000
So Sine Comp. $\odot\ Z\ D$, ———	48 8	9.82436
To Tang. $Z\ D$. ——— ———	31 11	9.78187

Then

Then by Concluſion the Seventh.

As Tang. —— ⊙ *P D*, 32 12 Co ar 0.20084
To Sine *Z D*; —— 31 11 —— 9.71414
So Tang. ⊙ *Z D*, —— 48 8 —— 10.04759

To Sine *P D*. —— 66 33 —— 9.96257

From *P D*, 66 33
Take *Z D*, 31 11

Reſts *P Z*, 35 22 the Complement of Latitude; hence the Latitude required is 54 38.

PROBLEM VI.

Plate 6. Fig. 3.

Given the Sun's Altitude, Azimuth, and Hour of the Day, to find the Angle of Poſition.

Altitude, 47 48
Azimuth, 131 52 } to find the Angle of Poſition,
Hour, — 32 32 } viz. the An. *Z* ⊙ *P*, by Ca. ſix.

Projected as in Prob. 4, the Perpendicular falls without from ⊙, as in Prob. 5. (See Plate 6, Fig. 4.) then in the Right-angled Triangle *Z D* ⊙, is given, as in Prob. 4, to find the Angle *Z* ⊙ *D*.

As Tang. Comp. ⊙ Z *D*, —— 48 8' 9.95240
To Radius; —— —— —— 10.00000
So Sine Comp. ⊙ Z —— —— 42 12 9.86970

To Tang. Comp. Z ⊙ *D*. —— 50 25 9.91730

Then by Concluſion the Fifth.

As Sine Comp. ⊙ Z *D*, — 48 8 Co ar 0.17561
To Sine of Z ⊙ *D*; —— 50 25 —— 9.88688
So Sine Comp. ⊙ *P D*, — 32 12 —— 9.92745

To Sine of *P* ⊙ *D*. —— 77 43 —— 9.98995

From the whole Angle *P* ⊙ *D*, 77 43
Take the Angle Z ⊙ *D*; —— 50 25

Reſt the Angle *P* ⊙ Z requir. 27 18 the Angle of Poſition.

Plate 7. *Fig.* 1.

PROBLEM VII.

Given the Latitude, Altitude and Azimuth, to find the Hour of the Day.

Latitude North, — 54 40
Altitude of the Sun, 47 48 } to fi. the H. of the
Azimuth, —— — 131 52 } D. by C. the Sev.

I ſhall project this upon the Plain of the Horizon, whoſe Pole or Center is the Zenith, the

the Point at which the given Angle falls; therefore,

P C Q, then is *n s* the Meridian of the Place. Draw the Azimuth *a z b*, to make with the Meridian, the given Angle, 131 52, at *z*, by Prob. 3, Caſe 1, of Spherical Geometry; then by Prob. 9, ſet off the Complement of Latitude, 35 20, from *Z* to *P*, and the Complement of Altitude from *Z* to ⊙, and through the Points *P* and ⊙, draw the Meridian *d P* ⊙ *c*, by Prob 5; and then you have finiſhed the Triangle *Z P* ⊙, in which there is given *P Z*, 35 20, and *Z* ⊙, 42 12, and the Angle *P Z* ⊙, 131 52, to find the Angle ⊙ *P Z*, by Caſe the Seventh.

Here by Solution the Ninth, the Perpendicular falls without, either from ⊙ or *P*: I ſhall inſtance in the Former, and then in the Right-angled Triangle, *Z e* ⊙, there is given the Angle ⊙ *Z e* (the Complement of ⊙ *Z P*, to 180) 48 8, and the Hipothenuſe ⊙ *Z*, 42 12, to find *Z* ⊙, by Caſe the Firſt.

As Tang. Comp. ⊙ *Z*, —— —	42 12		10.04251
To Radius; —— —— — ——			10.00000
So Sine Comp. ⊙ *Z e* —— —	48	8	9.82438
To Tang. *e Z*. —— ——	31	11	9.78187

Then by Concluſion the Seventh.

As Sine of the whole Baſe *e P*, } 66 31 Co ar 0.03754
To Tang. of the Angle *e Z* ⊙ ; } 48 8 — — 10.04759
So Sine of the Segment *e Z*, } 31 11 — — 9.71414

To Tang. of the Angle ⊙ *P e*. } 32 12 — — 9.79927

This 32 12, reduced to Time, gives 2 Hours 9 Minutes, the Hour from Noon required.

plate 7. *Fig.* 1. PROBLEM VIII.

Given the Latitude, Altitude, and Azimuth, to find the Declination.

Latitude North, 54 40
Altitude, ——— 47 48 } to find the Sun's Decl.
Azimuth, —— 131 52 } by Caſe the Eighth.

Having projected it, and let fall the Perpendicular, and found the Baſe *Z e*, as in Prob. 7, which is there found to be 31 11, proceed by Concluſion the Sixth, thus;

As Sine Comp. *Z e*, —— 31 11 Co ar 0.06777
To Sine Com. Z ⊙ ; —— 42 12 — — 9.86970
So Sine Comp. *P e*, — — 66 31 — — 9.60041

To Sine Comp. ⊙ *P*. —— 69 49 — — 9.53788

The

The Complement of Declination, 69 49, ſubſtracted from 90, gives 20 11, the Declination required.

PROBLEM IX. *plate 7. Fig. 3.*

Given the Latitude, Azimuth, and Hour, to find the Declination.

Latitude North, 54 40
Azimuth, —— 131 52 } to find the Sun's Dec.
Hour. —— 32 12 } by Caſe the Ninth.

To project this upon the Plain of the Meridian, becauſe the given Angles are at the Meridian, and the given Side is an Arch thereof.

P C Q, ſet the Complement of Latitude, 35 20, from *Z* to *P*, and draw *P S*, and at Right-angles, there to draw *E Q*; then by Prob. 3, Caſe 2, of Spherical Geometry, draw *P* ⊙ *S*, to make the given Angle, 32 12, with the Primitive at *P*, and (by the ſame) draw *Z* ⊙ *n*, to make the given Angle, 131.52, with the Primitive at *Z*; and where they interſect each other, as at ⊙, determines the Triangle, *Z P* ⊙, in which is given, *Z P*, 35 20, and *Z P* ⊙, 32 12, and ⊙ *Z P*, 131.53, to find ⊙ *P*, the Complement of Declination.

Here the Perpendicular (by Solution 9.) muſt fall upon the Side ⊙ *Z*, continued, which for Illuſtration I have annex'd to the Scheme; then in the Triangle *Z e P*, is given *e Z P*, 48 8, and *Z P*, 35 20, to find *Z P e*, by Caſe the Third, of Right-angled Spherical Triangles.

As Tang. Comp. *P Z e*,	48	8	9.95240
To Radius;			10.00000
So Sine Comp. *Z P*,	35	20	9.91158
To Tang. Comp. *Z P e*.	47	41	9.95918

Then by Concluſion the Eighth.

As Sine Comp. *e P* ⊙,	79	53	Co ar 0.75534
To Tang. *P Z*;	35	20	9.85059
So Sine Comp. *e P Z*,	47	41	9.82816
To Tang. *P* ⊙.	69	48	10.43409

The Complement of Declination being 69 48, the Declination required is 20 12.

Plate 7. *Fig.* 3.

PROBLEM X.

Given the Latitude, Azimuth, and Hour, to find the Angle of Poſition.

Latitude

Latitude North, — 54 40 }
Azimuth, —— — 131 52 } to find the Ang. of
Hour. —— —— 32 12 } Posit. by Case 10.

Proceed in Projection, and Calculation, as in Prob. 9, till you find the Angle Z P e, which is 47 41.

Then by Conclusion the Fifth.

As Sine of Z P e, — —— 47 41 Co ar 0.13109
To Sine Comp. P Z e; — 48 8 — — 9.82438
So Sine Comp. e P ☉, — 79 53 — — 2.99319

To Sine Comp. P ☉ e. — 27 19 — — 9.94866

This 27 19, is the Angle of Position required.

PROBLEM XI.

Plate 7. Fig. 2.

Given the Sun's Altitude, Declination, and the Latitude of the Place, to find the Hour of the Day.

Latitude North, 54 40 }
Declina. North, 20 12 } to find the Hour of the
Altitude. —— 47 48 } Day by Case the Elev.

Projection upon the Plain of the Meridian,

P C Q,

$P\ C\ Q$, ſet the Complement of Latitude, 35 10, from Z to P, and draw $P\ S$, and at Right-angles thereto, draw $E\ Q$, to repreſent the Equator; draw the Parallel of Declination $d\ e$, 20 12, from the Equator, and the Parallel of Altitude $b\ c$, 47 48, above the Horizon (by Prob. 7, Var. 2, of Spherical Geometry) and where thoſe Parallels interſect each other, (as in ⊙) is the Point the Sun *is* in at that Time; then through ⊙, draw the Meridian P ⊙ S, and the Azimuth Z ⊙ π, and the Triangle $Z\ P$ ⊙ is finiſhed, in which you have given, $Z\ P$, 35 20, Z ⊙, 42 12, and P ⊙, 69 48, to find $Z\ P$ ⊙, by Caſe the Eleventh, the Operation grounded upon Concluſion the Twelfth.

	Co ar
As Tang. ½ the Baſe P ⊙, ——	34.54 0.15638
To Tang. ½ Sum of the Sides $Z\ P$, and Z ⊙;	38.46 9.90475
So Tan. ½ Diff. of the ſame Sid.	3.26 8.77811
To Tan. ½ Diff. Segm. of the Baſ.	3.57 8.83924

To the half Baſe P ⊙, —— ——	34.54
Add ½ Diff. Segments; —— ——	3.57
Sum is the greater Baſe ⊙ e, — —	38.51
Difference is the leſſer Baſe $e\ P$. —	30 5.7

Then in the Right-angled Triangle $P\ e\ Z$, is given $P\ e$, 30.57, $P\ Z$, 35 20, to find the Angle P, by Caſe the Fourth.

As

As Radius, ——— 10.00000
To Tang. *P ⊙* ——— 30 57 9.77798
So Tang. Comp. *P Z*, ——— 35 20 10.14940

To Sine Comp. ∠ *P Z*. ——— 32 14 9.92731

The Angle required, 32 14, converted into Times, gives two Hours 9 Minutes, the Hour from Noon required.

But if the greateſt Angle (*viz.* the Azimuth) had been required; then by Axiom the Third, and the Rule deduced there-from.

Z P, ——— 35 20
Z ⊙, ——— 42.12
P ⊙, ——— 69.48

Sum, ——— 147.20
½ Sum, ——— 73.40
——— 69 48

Diff. ——— 3:52

∠ *P*, 35 20, Sine Co ar ——— 0.23782
Z ⊙, 42 12, Sine Co ar ——— 0.17281
½ Sum, 73 40, Sine ——— 9.98211
Diff. 3 52, Sine ——— 8.82888

Sum — 19 22162
Sine Comp. of 65 55 — 9.61081
65 55

Doubled is, — 131 10 the Azimuth required.

PRO-

Plate 7. *Fig.* 4.

PROBLEM XII.

Given the Azimuth, Hour, and Angle of Poſition, to find the Latitude.

Azimuth, ——— 131 52
Hour, ——— — 32 12 } to find the Latitude
Angle of Poſition. 27 19 } by Caſe the Eleven.

In the Projection of this, I ſhall not trouble the Learner with the common Way of turning the Angles into Sides; but lay it down in its proper Appearance, from what is given, ſuppoſing that to be much more inſtructive to the Learner, and to give him a better Idea of what he is going about; and ſhall project it upon the Plain of the Horizon.

Draw the Primitive Circle, and croſs it with the Diameter *n S*, to repreſent the Meridian; then is *Z* the Zenith: Draw the Azimuth *e g*, to make the given Angle 131.52, at 2, by Prob. 3, Caſe 1, of Spherical Geometry; now becauſe the Angle at *P* is 32 12, and the Angle at ⊙ is 27 19; alſo becauſe the Quantity of Angle is determined by the Diſtance of the Poles of thoſe Circles, that contain the Angles (by Prob. 9, of Spherical Geometry) it is evident, that the Pole of the Circle to be drawn, muſt be diſtant from the Pole of the Circle *n s*, equal

equal to the Angle at *P*, *viz.* 32 12, and from the Pole of the Circle *r z g*, equal to the Angle at ⊙, *viz.* 27 19; therefore draw the Parallel *h i*, diſtant from *w*, the Pole of *π P s*, 32 12, equal to the Angle at *P* (by Prob. 7, Caſe 2;) and (by the ſame) draw the Parallel *k m*, diſtant from *n*, 27 19, the Angle at ⊙, and where theſe Parallels cut each other, as in *o*, is the Pole of the Circle to be drawn; then through *o*, and the Center *z*, draw the Line or Circle *a o z b*, upon which (by Prob. 9, of Spherical Geometry) ſet off 90 Degrees from *o* to *r* (becauſe every Circle is 90 Degrees diſtant from its Pole, and through *r*, draw the Oblique Circle *c r d*, to cut the Primitive in the oppoſite Points *c* and *d*, it will cut *n z s* in *P*, and *i* ⊙ *g* in ⊙; and then is the Triangle *Z P* ⊙ finiſhed, and the Side *Z P* meaſured (by Prob. 8, Caſe 2, is 35 20, the Complement of Latitude required.

But becauſe it is more common for Learners, to project upon the Plain of the Meridian; I ſhall alſo give that Variety from the Rules and Reaſons before-mentioned. *Plate* 7. *Fig.* 5.

P C Q, draw *Z* ⊙ *n*, to make the given Angle 131 52, at *Z*, with the Primitive, and find its Pole *e*, at 27 19, from which, equal to the Angle at ⊙, draw the Parallel *c m n*, by Prob. 7, Caſe 3; and at 32 12, from the Pole of the Primitive (*a*) draw the Parallel *i c n*; and where the Parallels cut each other, as in *n* (or

d, if the Triangle was to be projected in the lower Part of the Scheme) is the Pole of the Circle to be drawn; ſet 90 Degrees from *r* to *n*, and draw the Circle *P r d*, to cut the Primitive in the oppoſite Points and *d*, and 'tis done.

The Calculation is the ſame in both Projections, *viz.* Take the Complement of the greateſt Angle to 180, add that and the two leſſer Angles together, and from the half Sum, ſubſtract the Angle oppoſite to the Side required, and proceed as in Prob. 11.

EXAMPLE.

The greateſt Angle,	131 52
Its Complement,	48 8
Angle *Z P* ⊙,	32 12
Angle *Z* ⊙*P*,	27 19
Sum,	107 39
Half Sum,	53 49½
Angle oppoſite to Side required,	27 19
Remainder,	26 30½

⊙ Z P, Co ar — — — — — — — 0.12801
Z P ⊙, Co ar — — — — — — — 0.27337
Half Sum, 53 49 ½ — — — — — 9.90699
Remainder, — — — — — — — 9.64965

19.95802
Sine Comp. 17.40 — — — — — 9.97901

Doubled, is 35 20, the Latitude, or Side Z P required.

Thus much for the Application of Oblique Spherical Triangles in Queſtions of Aſtronomy: I ſhall inſtance in one Queſtion more, *viz.* In a Quadrantal Triangle; and then ſuppoſe there will be Variety ſufficient (with due Application) for the Learner's Improvement in this Part of Practical Aſtronomy.

PROBLEM XIII.

The Sun's Declination, and two different Latitudes under the ſame Meridian, given to find what Hour of the Day the Sun's Altitude is equal in both Latitudes, and what is the Sun's Altitude that Time.

The two Lat. North, { 50 0 / 70 0 } to find as above.
Declination North — 23 30

PRO-

PROJECTION.

P C Q, with the Lines *P S*, and *E Q*; ſet the given Latitudes from *P* to 50, and from *P* to 70, and draw *H* 50, for the Horizon, to Latitude 50, and *H* 70, for the Horizon, to Latitude 70 (becauſe the two Places have different Horizons) draw *r o m*, equally diſtant from each Horizon, and draw the Parallel of Declination *t* ⊙ *u*, diſtant from *P* 69 48, equal to the Complement of Declination: Now it is evident, The Sun is ſome where in the Line *r o*, becauſe it is equally diſtant from both the Horizons, and it is in the Parallel *t u*, becauſe that is the Parallel of the given Declination; therefore it muſt be at the Interſection at the Point ⊙: Then from 50, ſet 90 Degrees of the Chords both Ways, to *z* and *N*, you have the Zenith and Nadir to the Horizon, *H* 50; ſet 90 of the Chords alſo both Ways, from 70 to *c* and *h*, which will find the Zenith and Nadir for Latitude 70 00; draw the two great Circles *z* ⊙ *N*, and *c* ⊙ *h*, they are Azimuths, to each Horizon, the Arches *C* ⊙, and *Z* ⊙ (being equal) is the Complement of the equal Altitudes in both Latitudes, and the Angle *P o* the Hour when the Sun's Altitude is equal in both the above-mentioned Latitudes.

Now in order to Calculation, there is given in the Quadrantal Triangle ⊙ *P O* (beſides the Quadrantal Side *P O*) the Side *P* ⊙, the Complement of Declination, 66 30, and the Angle

⊙ *O*

⊙ *O P*, 30 0 (being the Complement of the Middle between the two Latitudes (which here is 60) to a Right-angle) to find the Angle ⊙ *P O*, the Hour after 6 in the Morning, or before 6 at Night.

As Radius,			10.00000
To Tang. of ⊙ *o P*;		30 0	9.76144
So Tang. Comp. ⊙ *P*,		66 30	9.63830
To Sine ⊙ *p o*.		14 32	9.39974

Which converted into Time, gives 58 Minutes, *viz.* 58 Minutes past 6 in the Morning, or 2 Minutes past 5 in the Afternoon.

Then in the Triangle *c P* ⊙, there is given *c P* ⊙ (the Complement of ⊙ *P o*) 75 28, and the Side *c P* 20 0, and *P* ⊙ 66 30, to find *c* ⊙, by Case the Eighth, of Oblique Spherical Triangles, the Perpendicular (by Solution the Ninth) falling within, *viz.* From *c*, upon the Side ⊙ *P*, to the Point *n*.

As Tang. Comp. *c P*,		20 0	10.43893
To Radius;			10.00000
So Sine Comp. *c P n*,		75.28	9.99957
To Tang. *n P*.		5 13	9.96064

Then

Then by Conclusion the Sixth.

As Sine Comp. $\varkappa P$, — — — 5 13 0.00180
To Sine Comp. $Z P$ 5 — — — 20. 0 9.97298
So Sine Comp. $\odot \varkappa$, — — — 61 17 9.68167

To Sine Comp. $Z \odot$. — — 63 2 9.65645

The Complement of which, 26 58, is the Altitude required, and the Hour before found (2 Minutes past 5.) when the Altitude and Hour is the same in the two given Latitudes.

I need not repeat the Operation for the Triangle $Z P \odot$, because it is evident by Construction, That $Z \odot$ is equal to $c \odot$; nevertheless, for the more certain Proof of the Truth hereof, I shall invert the Question, and take the Altitude (now found) 26 58, the Declination 23 30, and the two given Latitudes 50 and 70, severally, to find the Hour by Case 12, of oblique Spherical Triangles; and first, For Latitude 50.

40 4 0.19193
66.30 0.03760
63 2 9.99818

169 32 9.56853

84 46 19.79624 Sum.
63 2 9.89812 Half Sum. Sine Comp. 37 44
37 44

21 44

Doubled, 75 28

Whose

Whoſe Complement 14 32, is the Hour from 6 required.

Then for Latitude 70.

20 0 0.46594
66 30 9,03760
63 2 9.98446

149 32 9.30826

74.46 19.79626 Sum.
63 2 9.89813 Half Sum. Sine Comp. 37 44
37 44

11 44

Doubled, 75.28

Whoſe Complement, 14 32, is the Hour from 6, agreeing with the Hour found, by the fore going Operations.

NOTE, This Problem may be ſolved by finding the Time of the Day when the Sun is in the Prime Vertical, or Eaſt or Weſt Azimuth, of Latitude 60 (the middle Latitude between 50 and 70) for then is the Altitude equal in Places under the ſame Meridian, and equally remote; therefore you have, as in Prob. 11. of Aſtronomy Right-angled, the Latitude of the Place, and the Sun's Declination given, to find what Hour and Minute the Sun will be due Eaſt or Weſt by the following Proportion.

As

As Radius,			10.00000
To Tang. Comp. Lat.	60	0	9.76144
So Tang. Declination,	23	30	9.63830
To Sine of the Hour.	14	32	9.39974

This 14 32, converted into Time, gives 58 Minutes paſt 6 in the Morning, or 2 Minutes paſt 5 in the Afternoon, agreeing exactly with the Operation in Page 141; only I thought fit to inſert the former Method, becauſe its Solution depends upon a Quadrantal Triangle.

CHAP.

CHAP. V.

Of the ANALEMMA.

SECT. I.

Of the Orthographick *Projection of the Sphere, by ſome call'd the* ANALEMMA.

ORTHOGRAPHICK Projection of the Sphere, ſuppoſeth the Eye perpendicular to the Plain of that great Circle, upon which it is projected, and at an infinite Diſtance from it, and is moſt commonly, and indeed, moſt intelligibly, perform'd upon the Plain of the Solſticial Colure, *viz.* the Meridian that interſects

the Ecliptick, in the Tropicks of Cancer and Capricorn: For the projecting and measuring the Parts of the Analemma, there is only Sines and Chords required; the Primitive Circle is measur'd by the Chords, the right Circles by the Sines, and the Parallels being first reduced to the Primitive, are also measured upon it by the Chords.

I have instanced in an Orthographick Projection of the Sphere for Latitude 54 40, North, Sun's present Declination 20 12, North his greatest Declination suppos'd to be 23 30.

Extend your Compasses to 60 of the Chords, and with one Foot in the Center *a*, describe the Circle *Z H N O*, and draw *Z N*, the prime Vertical, and *H O*, the Horizon; then set 54 40 (the given Latitude) from *O* to *P*, and from *Z* to *Æ*, and from *H* to *S*, and from *N* to *Q*; and draw *P S* the Axis of the World, and *Æ Q* the Equator; then because the Sun's greatest Declination is 23 30, take 23 30 off the Chords, and set from *Æ* to *E*, and from *Q* to *C*, and draw *E C* to represent the Ecliptick: Set off the Chord of 20 12 the Sun's present Declination, from *Æ* to *b*, and from *Q* to *c*, and draw the Line *b c*, which is the Parallel of Declination for 20 d. 12 m. North, *g i h* is a Parallel of Altitude.

Plate 9. Fig. 1.

Arches

Arches of the Primitive, meaſur'd upon the Chords.

O p is the Latitude 54 40,
P Z the Complement of Latitude 35 20,
Æ E the Sun's greateſt Declination 23 30,
Æ b the preſent Declination 20 12,
K O the Sun's Altitude at 6 a-clock 16 21.

Arches of right Circles meaſur'd on the Sines.

a e the Sun's preſent Declination 20 12,
a f the Sun's Amplitude 36 40, from the Eaſt,
a ☉ the Sun's Longitude 60 00,
a d the Sun's Altitude when Eaſt or Weſt 25 3,
a i the Sun's Altitude when in the Point ☉. 47 48,
a n (being equal to *e g*) is 16 21 the Sun's Altitude at 6.

Arches of leſſer Circles to be reduced to the Primitive, and meaſur'd by the Chords.

g ☉ the Sun's Azimuth from the South, } in the Point ☉.
☉ i the Sun's Azimuth from the East, }
i e the Hour from 6 that the Sun is due Eaſt 15 7,
n e the Sun's Azimuth from the Eaſt or Weſt at 6 12 1,
e f the Aſcenſional Difference 31 16,
☉ e the right Aſcenſion 57 48.

Before I proceed to any Aſtronomical Problems, it will firſt be neceſſary that the Learner underſtand how to reduce a leſſer, or Parallel Circle to the Primitive, ſo as any Part of the Parallel may be meaſur'd upon the Chords.

1. Take half the Parallel in your Compaſſes, and ſet off from the Center of the Primitive upon any right Circle, where there is moſt Room, or Conveniency; and where the other Foot of the Compaſſes fall, make a Mark.

2. Take in the Compaſſes, the Part of the Parallel to be meaſur'd; and with one Foot in the aforeſaid Mark, turn the other Foot about, and make an occult Arch.

3. Lay a Ruler from the Center of the Primitive, ſo as juſt to touch the Extremity of the ſaid Arch, and obſerve where the Ruler cuts the Primitive, making a Mark.

4. Extend your Compaſſes from the Mark upon the Primitive, to the Point where the right Circle, by which the Work was done, cuts the Primitive; that Extent meaſur'd on the Chords, gives the Quantity of the Arch of the Parallel requir'd.

EXAM-

EXAMPLE.

I would reduce the Parallel *k l*, to the Primitive, in order to meaſure the Arch *i l*, of the ſaid Parallel. *Plate* 9. *Fig.* 2.

Take the Extent *l k* in your Compaſſes, and ſet it from *a* to *e*, upon the right Circle *d a m*, then take *i l* in your Compaſſes, and with one Foot in *c*, deſcribe the Arch *g*; a Ruler laid from *a*, to touch the Extremity of the Arch *g*, will cut the Primitive in *e*, and the Arch *d e* meaſur'd on the Chords, gives the Quantity of the Arch *i l* requir'd.

NOTE, To ſet off any Number of Degrees upon a Parallel, or leſſer Circle, is only the Reverſe of this:

EXAMPLE.

It is required to ſet 40 Degrees from *r* upon the Parallel *r t*:

Take 40 of the Chords, and ſet from *m* to *s*, and draw the Line *a s*; ſet *r t* from *a* to *x*, then with one Foot of your Compaſſes in *x*, take the neareſt Diſtance to the Line *a s*, and with that Extent, and one Foot in *r*, the other will reach to *u*, and *r u* is 40 Degrees in that Parallel.

SECT. II.

Problems in Aſtronomy ſolved by the Orthographick Projection.

Plate 9. Fig. 1. **PROBLEM I.**

Given { The Sun's Long. 60 0 } Requ. Sun's
{ Greateſt Declin. 23 30 } preſent Dec.

I ſhall in this, as in the ſame Problem of Stereographick, project it for the Latitude 54 40, that the ſame Scheme may ſerve in other Caſes, where the Latitude is given or requir'd.

Having drawn the Axis of the World, the Prime Vertical, the Equator, Ecliptick and Horizon, as before directed; ſet the Sine of 60 00, the given Longitude from *a* to ☉, and from ☉ take the neareſt Diſtance to the Equinoctial Æ ♎, at which Diſtance draw the Parallel of Declination *b c*, it cuts the Primitive in *b* and *c*, the Arch Æ *b*; or ♎ *c* meaſur'd on the Chords, or *a e* meaſur'd on the Sines, gives 20 12 the Declination requir'd.

Plate 9. Fig. 1. **PROBLEM II.**

Given { Sun's Longitude 60 0 } Requ. right
{ Greateſt Declin. 23 30 } Aſcenſion.

In the Scheme as projected before the Arch *e* ☉ of the Parallel of Declination *b c*, reduced to

to the Primitive as before directed, gives 57 48 the right Aſcenſion requir'd.

NOTE, The right Aſcenſion is properly an Arch of the Equator, intercepted between the neareſt Equinoctial Point, and the Point where a Meridian drawn through ☉, cuts the Equator; but in this Projection, the Meridians between the Primitive and its Center, not being Circles, or Parts of Circles, are not to be attain'd but by Approximation, *viz.* by finding ſeveral Points through which they are to be drawn, which takes up a great deal of Time, and renders the Work very troubleſome; and this Way of Reckoning the right Aſcenſion upon the Parallel being exact, and much more eaſy, I have rather made Choice of it the ſame Way, being equally uſeful in other Caſes.

PROBLEM III. *Plate 9. Fig. 1.*

Given { ☉'s greateſt Dec. 23 30 } to find Lon.
{ ☉'s preſent Dec. 20 12 } right Aſcen.

Draw the Primitive, the Axis, Horizon, Equator, Prime Vertical, &c. as before taught, and ſet 23 30 the greateſt Declination from Æ to *E*, and from ♑ to *C*, and draw *E C* the Ecliptick; ſet the preſent Declination 20 12 from Æ to *b*, and from ♑ to *c*, and draw the Parallel of Declination *b c*, and where that cuts the Ecliptick, as in ☉, is the Point the Sun is

in, and the Arch of the Ecliptick *a* ☉, measured on the Sines, is 60 Degrees the Longitude of the Sun required, and the Arch *e* ☉ 57 48, is the right Aſcenſion.

Plate 9. *Fig.* 1.

PROBLEM IV.

From the Scheme projected according to the Directions given in the Beginning hereof, for Latitude 54 40, and Sun's Declination 20 12, to find,

1. The aſcenſional Difference.
2. The Sun's Amplitude.
3. What Hour and Minute the Sun will be due East or Weſt.
4. Sun's Altitude at 6.
5. Sun's Altitude when East or Weſt.
6. Sun's Azimuth at 6.

1. The Arch ☉ *e* of the Parallel *b c*, measured the fore-going Rules, is 57 48 the aſcenſional Difference; the Uſe of which, for finding the Time of Sun's riſing and ſetting, and the Length of the Day or Night. See Prob. 8. and 9. of Aſtronomy.

2. The Arch *a f* of the Horizon meaſur'd on the Sines, is 36 40 the Sun's Amplitude from the East or Weſt.

3. The

3. The Arch *d e* of the Parallel *b c*, is 15 7, which reduced to Time, gives one Hour after 6 in the Morning, or one Hour before 6 in the Evening, *viz.* 7 in the Morning, or 5 in the Afternoon.

4. For the Sun's Altitude at 6, it ſhould be repreſented by *e q*, which is the Arch of an Azimuth; but there being great Difficulty, both in projecting and meaſuring the Azimuths in this Projection, and the Arch *n a* of the prime Vertical being equal to *e q*, (becauſe between the ſame Parallels, and cutting them at right Angles,) *n a* meaſur'd on the Sines, is 16 21 the Sun's Altitude at 6 required.

5. The Sun's Altitude when Eaſt or Weſt, is repreſented by *n d*, which meaſur'd on the Sines, is 25 Degrees 3 Minutes.

6. The Arch *n e* of the Parallel of Altitude *k l*, is 12 Degrees 1 Minute, the Sun's Azimuth at 6, reckoned from the Eaſt or Weſt Northerly, and its Complement *e k*, is the Azimuth from the North.

PROBLEM V. *Plate 9. Fig. 1.*

If at Sea you cannot have an Obſervation for the Latitude at Noon, but by the Benefit of a clear Evening, having the Declination given you,

you can find the Amplitude; and by the Help of that, and the given Declination, you deſire to know your Latitude.

Suppoſe the Amplitude at Sun's ſetting 36 40 from the Weſt Northerly, and the Declination 20 12 North to find the Latitude.

With the Chord of 60, draw the primitive Circle, and croſs it at right Angles, with *Z N* the prime Vertical, and *H O* the Horizon; ſet the Sine of 36 40 the given Amplitude from *a* to *f*, then take in your Compaſſes the Sine of the Declination 20 12, and with one Foot in *f*, ſweep the ſmall Arch *r*; then through the Center *a*, and by the Extremity of the Arch *r*, draw *E a Q*, which repreſents the Equator; and at right Angles thereto, draw *P S*; then the Arch *P O* meaſur'd on the Chords is 54 40 the Latitude ſought.

Plate 9. *Fig.* 1.

PROBLEM VI.

Or if you cannot get the Sun's Altitude at Noon, but have an Opportunity to obtain the Sun's Altitude at 6, either Morning or Evening, which ſuppoſe to be 16 21, and the Declination 20 12 North, to find the Latitude.

Having drawn the primitive Circle, and croſs'd it with the Diameters, *Z N* the prime Vertical, and *H O* the Horizon; ſet the Chord of

of 16 21 the given Altitude from *H* to *l*, and from *O* to *k*, and draw the Parallel of Altitude *l k*; then take in your Compaſſes the Sine of the given Declination 20 12, and with one Foot in *a*, with the other croſs the Parallel *l k*, in *e*, through *e*, and the Center *a*, draw the Line *P e a S*; the Arch *P O* meaſur'd on the Chords is 54 40 the Latitude required.

PROBLEM VII.

Plate 9. Fig. 1.

Given the Sun's Declination 20 12 North, and his Altitude when Eaſt or Weſt 25 3, to find the Latitude.

Draw the Primitive, and croſs it as before; ſet the Sine of the Sun's Altitude when Eaſt or Weſt, *viz.* the Sine of 25 3 from *a* to *d*; then with the Sine of 20 12 the Declination, and one Foot in *a*, ſweep the ſmall Arch *e*; then by the Extremity of that Arch, and through the Point *d*, draw the Parallel of Declination *b c*, and parallel thereto through the Point *d*, draw the Equinoctial ♈ *u* ♎; and at right Angles to it draw the Axis *P a S*; the Arch *P O* meaſur'd on the Chords gives 54 40 the Latitude.

Plate 9. *Fig.* 1. **PROBLEM VIII.**

You may alſo from this Projection ſolve thoſe Problems that depend upon Oblique Spherical Triangles; as ſuppoſe there is

Given { Sun's Altitude, 47 48 / Declin. North, 20 12 / Azi. from Eaſt, 41 52 } to find the Latit. and Hour.

Draw the Primitive, and croſs it with the prime Vertical and Horizon, as before, and ſet the Chord of the Altitude 47 48 from *H* to *g*, and from *O* to *h*, and draw *g h* for the Parallel of Altitude; then by the Directions given for meaſuring and ſetting of Degrees upon a Parallel, ſet 41 52 the given Azimuth from *i* to ⊙, and with the Sine of the given Declination, and one Foot in ⊙, make the ſmall Arch *x*, through the Extremity of which, and the Center *a*, draw *Æ a Q* the Equinoctial, (becauſe the Sun's Declination is his neareſt Diſtance from the Equinoctial,) and parallel thereto, through the Point ⊙, draw the Parallel of Declination *b e c*, alſo at right Angles with the Equinoctial, draw the Axis *P a S*; the Arch *P O* is the Latitude ſought, and the Arch ⊙ *e* of the Parallel *b c* is 57 48 the Hour from 6, which allowing 15 Degrees to an Hour, and one Degree to 4 Minutes, gives 3 Hours 51 Minutes after 6 in the Morning, or before 6 in the

the Evening, *viz.* 51 Minutes paſt 9 in the Morning, or nine Minutes paſt two in the Afternoon.

PROBLEM IX.

Plate 9. Fig. 1.

Given { ☉'s Altitude, — 47 48 } to find the
{ ☉'s Declin. North, 20 12 } Azimuth
{ Latitude North. — 54 40 } and Hour.

Draw the Primitive, and croſs it with the Horizon *H O*, and the prime Vertical *Z N*; ſet the Latitude 54 40 from *O* to *P*, and draw *P S*, and at right Angles thereto draw the Equinoctial *Æ Q*; ſet the Chord of Declination 20 12 from *Q* to *c*, and from *Æ* to *b*, and draw the Parallel of Declination *b c*; likewiſe ſet the Altitude 47 48 from *H* to *g*, and from *O* to *h*, and draw the Parallel of Altitude *g h*; and where that cuts the Parallel of Declination, as in the Point ☉, is the Place the Sun is in; and ☉ *i* 41 52, is the Sun's Azimuth from the Eaſt or Weſt; and ☉ *e* 57 48 is the Hour, as in Prob. 8.

I might have inſtanced in more Problems of this kind, but I think what I have already done, is ſufficient to inſtruct the Learner in the Nature and Uſe of this Projection.

CHAP.

CHAP. VI.

Of DIALLING.

SECT. I.

How to project a DIAL *Stereographically upon any Plain.*

THE Projection of any Plain Dial, is no more but the Projection of the Sphere, upon the Plain of that great Circle, to which the Dial is Parallel, the Meridians at every 15 Degrees Diſtance repreſenting the Hour Lines, and the Axis of the World (terminating at the two Poles) repreſenting the Stile; as the following Examples will make evident.

SECT.

SECT. II.

How to project an Horizontal DIAL.

AN Horizontal Dial, is ſo called, becauſe it is parallel to the Horizon, and therefore is projected upon the Plain thereof, to the Latitude given.

EXAMPLE.

It is required to draw an Horizontal Dial for the Latitude of *London*, 51 Degrees 32 Minutes.

Firſt, Draw the Primitive Circle *N W S E*, to repreſent the Horizon, and croſs it at Right-angles, with the Lines *N S*, and *W E*, then the Center *Z* is the Zenith of the Place (becauſe it is the Pole of the Horizon) and the Points *N*, *S*, *E*, *W*, are the North, South, East and Weſt Points of the Projection.

Then becauſe the Pole of the World is diſtant from the Zenith 38° 28′ (equal to the Complement of Latitude) ſet off 38° 28′ upon the Right Circle *S N*, by Prob. 9. from *Z* to *P*; then is *P* the Pole: alſo becauſe the Equator is diſtant from the Zenith 51° 32′ the Latitude

titude given, ſet off 51° 32′ from *Z* to *Q*, by Prob. 9. of Spherical Geometry; and draw the Oblique Circle *W Q E*, to repreſent the Equator; then from *Q* to *a*, ſet off 15 Degrees each Way upon the Equator, by Prob. 9. and from *Q* to *b*, 30 Degrees each Way, and from *Q* to *c*, 45 Degrees, and to *d*, 60 Degrees, and to *e*, 75, as in the Figure; then through *P*, and the ſeveral Points *a b c*, &c. draw the Meridians 1 *P a* 1, and 2 *P b* 2, *&c.* by Prob. 5; which done, draw Lines from the Center *Z*, to the Points 1, 2, 3, *&c.* theſe are the Hour Lines required.

Then for the Stile's Height, meaſure the Arch *P N*, of the Right Circle *S N*, by Prob. 8. the Quantity of that Arch is the Height of the Stile required.

NOTE, If you draw your Projection upon the ſame Plain (whether Wood, Stone, *&c.*) that you intend to make your Dial upon, 'tis beſt to do it obſcurely, that when you have found the Hours Diſtances upon the Periphery, the other Circles, *&c.* may be done out at Pleaſure, and the Stile may be laid down, by ſeting 51° 32′ of the Chords from *N* to *B*; then draw the Line *Z R*, and *R N*; but you may make the Lines *Z R*, and *Z N*, longer, or ſhorter, at Pleaſure; only the Angle at *Z* is to be always equal to the Latitude of the Place: Thus a Piece of thin Plate, *&c.* of the Form of the Triangle *Z R N*, ſhall be the Stile required.

To

To find the Hours Diſtances by Trigonometrical Calculation, the Proportion is,

As Radius,
To Sine of Latiude;
So Tangent of 15 Degrees,
To Tangent of the Hours Diſtance of 11 and 1 from the Meridian:

And ſo Tangent of 30 Degrees,
To Tang. of the Hours Diſtance of 10 and 2.
And ſo of 45 Degrees,
For 9 and 3, *&c.*

This Proportion you have in many Books of Dialling; but no Reaſon given for it, nor Demonſtration of it, as you have from this Projection, in which, in the Triangle *P N* 1, you have given *P N*, the Latitude 51 32; and the Angle at *P*, 15 Degrees, to find the Side *N* 1, the Diſtance upon the Primitive Circle, between the Hours of 12 and 1 (or 12 and 11, which is the ſame;) and you may find it by the Globical Projection, taught in the former Part of this Book; for *Plate* 8. *Fig.* 3. ſuppoſing the Triangle *P N* 1, to repreſent the Triangle *P N* 1 in the Projection, you will find by the Rule laid down in Globical Projection, that the Proportion lies in the Manſion *P b c*, and is,

As *(P b)* Radius,	90.00	10.00000.
To *(P n)* the Sine of Latitude 51 32		9.89374.
So *(b c)* the Tangent of 15 Degrees the Angle at *P*,		9.42805.
To *(N 1)* the Distance upon the Primitive Circle between the Hr. of 12 and 1; *viz.* 11° 51′, whose Tangent is,		9.32179.

By the same Proportion with Tangent of 30 Degrees, you find the Hour Distances of 10 and 2, and by 45 Degrees, the Hour of 9 and 3, *&c.* which is an intelligible Demonstration of that common given Proportion above mentioned.

SECT. III.

How to project an Erect Direct South DIAL.

THIS Dial is parallel to (and therefore projected upon the Plain of) the Prime Vertical, or East and West Azimuth.

EXAMPLE.

'Tis required to draw a South Dial for the Latitude of 55° 8′.

First, Draw the Primitive Circle *W Z E N*, to represent the Prime Vertical; then cross it at Right-angles, with the Line *Z N* (which in this Case is the Meridian of the Place) and *E W*, the Horizon; then is *Z* the Zenith, *N* the Nadir, *E* East, and *W* the West Points of the Projection; then because the Equator is 34° 52′ above the Horizon, equal to the Complement of Latitude, set off 34° 52′, from *S* to *Q*, by Prob. 9. and draw the Oblique Circle *W Q E*, to represent the Equator: Also because the South Pole (which in this Case is the visible Pole) is below the Horizon 55° 8′, equal to the Latitude, set off 55° 8′, from *S* to *P*, by Prob. 9. then is *P* the South Pole; this done, set off upon the Equator 15:30:45 Degrees, *&c.* both Ways, from *Q* to *a b c*, &c. and draw the Meridians 1 *P a* 1, 2 *P b* 2, *&c.* to cut the Primitive Circle in the Points 1 2 3, *&c.* then Lines drawn from the Center *S*, to the Points 1 2 3, are the Hour Lines required.

The Height of the Stile is always equal to the Complement of Latitude, which here is 34° 52′, and may be made as directed in the Horizontal Dial, the Angular Point of the Stile both in this and Horizontal, being

ing fixed at the Center of the Dial, and the bottom of the Stile proceeding upon the Meridian Line towards the Hour of 12.

A North Dial is only projected on the contrary Side of the Prime Vertical; but the Hour-Diſtances, Height of the Stile, *&c.* is all the ſame; only in a North Dial, the Hour of 12 is put upwards, and the Angular Point of the Stile is put downwards; and therefore to avoid Prolixity, I ſhall ſay no more about the Projection of it.

The Proportion for finding the Hour Diſtances on a South Dial, by Trigonometry, is,

As Radius,
To Sine Complement of Latitude;
So Tangent of 15 Degrees,
To Tangent of the Diſtances of the Hours of 11 and 1 from the Meridian;

And ſo by the Tangent of 30 Degrees, 45 Degrees, 60 Degrees, *&c.* you find the Hour Diſtances of 10 and 2, of 9 and 3, and of 8 and 4, *&c.* by the Rules laid down in the Horizontal Dial, by which you may alſo find the Reaſon of the Proportion in a South Dial, in the Triangle *P N* 1, by the Directions before-mentioned.

SECT.

SECT. IV.

How to make an Eaſt or Weſt DIAL.

BEFORE you can project this Dial, you muſt be determined how high your Stile ſhall be, which may be done at Pleaſure, according to the Largeneſs of your Plate, or Plain, which is moſt proper to be an Oblong, or long Square, upon which draw a Line right-up the Middle, as *a b*; then croſs it at Right-angles, with the Line *c d*; which done, take in your Compaſſes the Height of your Stile, and with one Foot in the Croſſing of the aforeſaid Lines, with the other ſweep the Circle *c e d h*, which divide into 24 equal Parts, and draw the two Tangent Lines *k m*, and *n o*, Parallel to the Line *a b*, and juſt to touch the Circle *c e d h*, in *c* and *d*; then is the Line *c d* the Hour Line of 6, upon which the Stile muſt ſtand; then from the Center *q*, lay a Scale over each Diviſion of the Circle, and obſerve where it cuts the Lines *k m*, and *n o*; and from theſe Marks, draw the Lines 5, 5, 7, 7, *&c.* which ſhall be the Hour Lines required. *Plate* 8. *Fig.* 4.

The

The Dial thus finiſhed, is fit for any Latitude; only when you put it up, you muſt have it ſo ſituate, that the Line *c d* may juſt point to the Poles, then will *a b* be Parallel to the Equinoctial; and ſo place it upright upon the Eaſt Side of a Wall, *&c.* the Height of the Stile equal to the Semidiameter of the Circle by which it is projected; and then is your Dial ready for Uſe.

A Weſt Dial is the ſame; only inſtead of 5, 6, 7, 8, *&c.* 'tis numbred 8, 7, 6, 5, *&c.* but always the 6 a Clock Line is that that cuts the Circle in 2 equal Parts, and paſſes over its Center.

The Trigonometrical Proportion for finding the Hour Diſtance, is,

As Radius,
To the Height of the Stile;
So Tangent of 15 Degrees,
To the Diſtance 6 7;

And ſo Tangent of 30 Degrees,
To Diſtance 6 8;
And ſo Tangent 45 Degrees, or Radius,
To Diſtance 6 9, *&c.*

Hence the Hour Line of 9, ſhould always juſt touch the Circle; becauſe Tangent 45 Degrees is equal to Radius; and the Reaſon of this Proportion is ſo apparent to any that underſtand

derſtand the Conſtruction of the Tangents, that it is needleſs to inſiſt any more upon it.

SECT. V.

How to project a Declining DIAL.

IT is required to project a Dial for Latitude, 51 32, declining from the South Weſtward 30 Degrees.

Firſt, Draw the Primitive Circle *b c d e*, and croſs it with two Diameters *b d* and *c e*; then becauſe the Declination is Weſt 30 Degrees, draw the Oblique Circle *b n d*, to repreſent the Meridian of the Place, to make an Angle of 30 Degrees, with the right Circle *b a d* (the Meridian of the Plain (by Prob. 3. Caſe 2.) and becauſe the Pole of the World is always in the Meridian of the Place *b n e*, and diſtant from the Horizon (*viz.* below it) equal to the Latitude; draw the Parallel (*h i k*) diſtant from the Horizon (*c a e*) 51 32, and where it cuts the Meridian *b n d* (as in *m*) is the Pole of the World; through *m* draw *a m g*, and now you have the two Right-angled Triangles *m g d*, and *m n a*, and the Quadrantal Triangle *a m d*,

m, *d*, to find the Requiſites in your Dial, and the Reaſon of the Proportions uſed for that End, for the ſeveral Arches and Angles in theſe three Triangles, meaſur'd by the Rules laid down in the Geometrical Part hereof, gives the Quantity of the Requiſites by them repreſented; for the Circle *b n d* repreſents the Meridian of the Place, *b a d* the Meridian of the Plain, *a m g* the Subtile, *m g* the Stile's Height, *a i* the Latitude, *&c.* and the Angle *i d m*, the Declination of the Plain *m d g*; its Complement *d g* (the Meaſure of the Angle *i a m*) is the Subtile's Diſtance from the Meridian *a n* (the Meaſure of *a d n*) the Declination, *&c.*

Then firſt, For the Subſtile's Diſtance from the Meridian, repreſented by the Side *d g*, in the Triangle *m g d*, in which you have given *m g d* the Complement of Declination, and *m d* the Complement of Latitude to find *d g*, the Subtile's Diſtance from the Meridian, which by my Lord *Nepair*'s Proportion, is,

As Tangent Latitude, — ——	51:32	10.09991
To Radius; ——— —— —	90:00	10.00000
So Sine Declination, —— ——	30:00	9.69897
To Tangent Subſ. Diſt. ———	21:40	9.59906

Or in the Quadrantal Triangle *a m d*, the Angle *d a m*; or in the Triangle *m n a*, the Complement of the Angle at *a* is the ſame; but ſhall leave the Operations for the Reader's Practice.

Secondly,

Secondly, For the Stile's Height in the Triangle *m g d*, the Side *m g* is required from the former Data, and is by the Gnomonical Projection,

As Radius, ——— — ———	90: 0:	1.000000
To Sine Complement Dec. S. C.	30: 0.	9.93753
So Sine Complement Lat. S. C.	51:32	9.79383
To Sine of the Stile's Height.—	32 56	9.73136

Or in the Quadrantal *a m d*, or the Right-angled Triangle *m n a*, the Side *a m* common to both, is the Complement of the Stile's Height required.

Thirdly, For the Difference in Time between the Meridian and Subſtile, which is the Angle *d m g*, it is found by the Globical Projection to be,

As Sine Latitude, ——— ——	51:32	9.89374
To Radius; —— —— ———	90:00	10.00000
So Tangent of Declination, —	30:00	9.76144
To Tang. of Diff. in Time, *&c.*	36 24	9.86770

Which 36 24, converted into Time, is 2 Hours 26 Minutes, between the Meridian of the Plain, and the Subſtile; and if you draw all the Meridians through the Pole *m*, and making an Angle with each other of 15 Degrees, the Points where they interſect being mark'd with 12:1:2:3, *&c.* and Lines drawn from the Center *a* to each Mark, ſhall be the Hour Lines required.

I have not drawn all the Meridians, leſt it ſhould too much confuſe the Diagram; but I have drawn the Hour Line, of 6 (*viz.* 6 *m.* 6.) by which you may do all the reſt by Prob. 3. Caſe 4. Var. 2. of Spherical Trigonometry, making an Angle of 15 Degrees with each other at the Pole *m*.

Thus you have not only the Proportions given, which you meet with in moſt Books of Dialling; but alſo the Reaſons of them, as deduced from the known Principles of Spherical Trigonometry, and Projection of the Sphere, the true Foundation of which being known, 'tis very eaſily apply'd to any Caſe, either in Aſtronomy, Dialling, or Geography, or any Part of the Mathematicks, that depend upon Spherical Triangles.

SECT. VI.

How to draw a South DIAL, *or a Horizontal* DIAL *upon their reſpective Plains without the Help of any Mathematical Calculation.*

1. A Dial parallel to the Plain of the Equinoctial, is only a Circle divided into 24 equal Parts for the Hours; and the Stile or Axis is erected from the Center perpendicular to the Plain.

2. The

2. The Plain of an Equinoctial Dial, makes an Angle with the Plain of a South Dial, for any given Latitude equal to the ſaid Latitude; and with a Horizontal Dial, the Equinoctial Dial makes an Angle equal to the Complement of the Latitude, for which the Equinoctial Dial is made.

3. All Dials, however ſituate, have their Stiles parallel to each other (becauſe all parallel to the Axis of the World) conſequently if a South Dial was erected upon the North End of a Horizontal, one Stile would ſerve for both, making an Angle with the Horizontal Dial equal to the Latitude, and with the South Dial equal to the Complement of the Latitude of the Place, the two Plains making a right Angle with each other.

Hence alſo if a South Dial was erected at the North End of an Equinoctial Dial in North Latitude (or a North Dial at the South End in South Latitude) and contiguous to it, one Stile would likewiſe ſerve for both, making right Angles with the Plain of the Equinoctial Dial, and an Angle equal to the Complement of Latitude with the Plain of the South Dial (the two Plains making an Angle with each other equal to the Latitude.)

From theſe Conſiderations, a general Rule may be deduc'd, for transferring the Hour Lines from an Equinoctial Dial, to a South Dial, or to a Horizontal Dial, the equal Diviſions in one becoming unequal in the other; as for Inſtance,

Plate 10. *Fig.* 3. It is required to make a South Dial for Latitude 40 Degrees North.

First, Draw the Line *e c*, ſuppos'd Perpendicular to the Horizon, to repreſent the Plain upon which the South Dial is to be made; then from *c*, draw the Line *c a*, to make Angle of 40 Degrees with the Line *c e* (becauſe the Plain of the Equinoctial Dial makes an Angle of 40 Degrees with the Plain of the South Dial) then from any convenient Point in the Line *a c*, erect the Perpendicular *b d*, to repreſent the Stile (becauſe the Stile is Perpendicular to the Plain of an Equinoctial Dial) and that will alſo repreſent the Stile of the South Dial, making an Angle with the Plain of it at *d*, equal to the Complement of Latitude, *viz.* 50 Degrees. This done with the Extent *b c*, and one Foot in the Center *n*, deſcribe the Circle *g h* 12 9, which divide into 24 equal Parts in the Points 12 1 2 3, *&c.* Continue the Line *g n* 12 to *l*, and at right Angles therewith, draw the Line *o p*, ſo as juſt to touch the Circle in the Point 12, and produce the Lines *n* 4, *n* 3, *n* 2, *&c.* to cut the ſaid Line in *o r s*, &c. then from the Line *c e* (*Fig.* 3.) which repreſents the Plain of the South Dial, take the Extent *d c* (which repreſents the Semidiameter of the South Dial) and with one Foot in the Point 12, where the Line *o p* cuts the Line *g l*, with the other find the Point *q*, upon which as a Center with the ſame Extent *q* 12, deſcribe the Circle 12 *k l m*, and from the Points *r s t*, &c. draw the Lines *r q*, *s q*, *t q*, &c. and where

Plate 10. *Fig.* 1.

where theſe cut the Circle 12 *k l m*, as in 4 3 2 1, *&c.* are the unequal Diviſions repreſenting the true Hour Lines upon the South Dial requir'd, the Stile fixed at the Point *q* making an Angle of 50 Degrees with the Plain of the Dial equal to the Complement of the propos'd Latitude.

The Reaſon of this Method may yet be better conceiv'd, if you imagine this Paper upon which the Projection is drawn, to be folded in the Line *o p*, ſo that the Plain *g* 12 may make an Angle of 40 Degrees (the given Latitude) with the Plain 12 *l*; for then the Perpendicular rais'd from *n* the Center of the leſſer Circle, will fall in *q* the Center of the greater Circle, as is evident from the Figure *a b c d e*, Fig. 3.

For thoſe that underſtand Trigonometry, obſerve in the Triangle *b c d*, (*Fig.* 3.) the Angle *c* is the Angle that the two Plains make with each ther, equal to the Latitude, and conſequently the Angle *d* is equal to the Complement of Latitude; becauſe the Angle *b* is a right Angle. The Side *b c*, is the Semidiameter of the Equinoctial Dial, which may be aſſumed of any Length at Pleaſure, and *d c* is the Semidiameter of the South Dial; for the finding of which, the following Proportion is deduced from the ſaid Triangle.

As Sine of the Angle *b d c*,
To Side *b c*;
So Radius,
To Side *d c*.

Therefore,

As Sine Complement of Latitude,
To Radius;
So the Semidiam. of the Equinoctial Dial,
To the Semidiameter of the South Dial.

And in this, as in all other Cases, having found the Hour Lines, you may put the Center *q* to the upper Part of the Dial, and reduce it to a Square, or an Oval, *&c.* and add to it what Ornament you think fit.

NOTE, Always take Care to give Allowance for the Thickness, or Substance of your Stile, by imagining the Line upon which it stands to be split down the Middle, and the Parts removed asunder, as far as the Thickness of the Stile; for if you lay the Meridian, or Subtile only under the Middle of the Stile, your Dial will be too slow when the Sun is on the East Side of the Stile, and too fast when on the West Side, sometimes by a Quantity of the Shadow (nearly) equal to half the Thickness of the Stile.

The Horizontal Dial may by the same Means be projected from the Equinoctial Dial; as suppose I would make a Horizontal Dial, for the Latitude of 40 Degrees, having drawn the Line ☽ *c*, to represent the Plain parallel to the Horizon; then from *c* draw the Line *c a*, to make with the Line ☽ *c*, an Angle of 50 Degrees at *c*, equal to the Complement of Latitude; then from any Point at *b*, in the Line *a c*, draw the Line *b* ☉, perpendicular to *a c*; then is *b* ☉ the Stile to both Dials, *viz.* the Equinoctial

Plate 10. *Fig.* 2.

quinoctial Dial *a c*, and the Horizontal Dial ☽ *c*; then with the Extent *b c*, draw *a* Circle, which divide as before, into 24 equal Divisions; and upon the 12 a Clock Line, continu'd with the Extent ⊙ *c*, and one Foot in 12, find the Center of the greater Circle, and with the same Extent draw the Circle to represent the horizontal Dial, and so proceed in all respects as before, to transfer the equal Divisions in the lesser Circle to the tangent Line, and from thence to the greater Circle's Center, and they will make the unequal Divisions upon the Periphery of the greater Circle, which shall describe the Hour Lines requir'd, which being done in all respects, as in the South Dial, I shall not need to make any further Repetition.

The trigonometrical Proportion for finding the Diameters of the greater Circle, deduc'd from the Triangle ⊙ *b c*, is;

As Sine of *b* ⊙ *c*,
To Radius;
So the Side *b c*,
To the Side ⊙ *c*.

That is,

As Sine of Latitude,
To Radius;
So the Semidiameter of the lesser Circle,
To the Semidiameter of the greater.

NOTE, You need not project any more than 6 Hours, *viz.* from 12 to 6; for Hours equally distant from 12, are equally distant from one another, *viz.* the Distance between

the Hour Lines of 9 and 10, is equal to the Diſtance between thoſe of 2 and 3, and the ſame of thoſe equally diſtant from 6, as from 7 to 8, is the ſame Diſtance as from 4 to 5, and the oppoſite Hours are alſo equally diſtant, as from 4 to 5 in the Morning, is the ſame Diſtance upon the Circle, as between 4 and 5 in the Afternoon.

CHAP. VII.

Of GEOGRAPHY.

THE principal Subject of this Treatiſe being Trigonometry, and the Application thereof to Practice, my Intent in this Chapter, is to ſhew the Uſe of Spherical Trigonometry, when apply'd to Problems in Geography; in which I ſhall not have Occaſion for many.

DEFINITIONS.

Latitude of a Place, is an Arch of the Meridian, intercepted between the ſaid Place, and the Equinoctial; and therefore may be either North or South.

Longitude of a Place, is an Arch of the Equinoctial, contain'd between the firſt Meridian, and the Meridian of the Place; and is either Eaſt or Weſt.

NOTE, The firſt Meridian, is any Meridian made Choice of to reckon Longitude from; but our *Engliſh* Geographers of late, generally reckon their Longitude from the Meridian of *London*.

Di-

Diſtance between two Places, is twofold, *viz.* in a Rumb, or in the Arch of a great Circle; but the latter being the neareſt, and trueſt, is what I ſhall give ſome general Directions for at this Time.

Finding the Diſtance between any two Places, is commonly expreſs'd under three Varieties, *viz.* 1. When Places differ only in Latitude. 2. When they differ only in Longitude. 3. When they differ both in Longitude and Latitude; but I ſhall reduce all theſe Varieties to this one

GENERAL RULE.

Having the Latitude and Difference of Longitude, of two Places given, find the Points repreſenting them in the Stereographick Projection; then through the ſaid Points draw a great Circle by Prop. 5. of Spherical Geometry; that Arch (contain'd between the ſaid Points) meaſur'd by Prob. 8. of ſpherical Geometry, gives the Diſtance requir'd, allowing every Minute a Mile, and every Degree 60 Miles.

EXAMPLE. *Plate* 10. *Fig.* 4.

Suppoſe two Places, both in Latitude 40 Degrees North, Difference of Longitude 38 Deg.

P C Q, then is *P* the North, *S* the South Pole, *E n Q* the Equinoctial; draw the Parallel of 40 Degrees *a b c*, aſſume any Point therein, as *d*, for one of the two Places; through *d* draw the Meridian *P d S*, and obſerve where it cuts the Equinoctial, as in *o*; from *o* ſet off

the Difference of Longitude, 38 Degrees, to *q*, and draw the Meridian *P q S*, it will cut the Parallel *a b c*; in *c*, through *d* and *c*, draw the great Circle *R d c T* the Arch *d c* measur'd by Prob. 8. of ſpherical Geometry, gives the Diſtance requir'd.

CALCULATION.

The two Latitudes being equal, you have an Iſoſcelis Triangle *P d c*, in which is given *P c*, equal to *P d*, 50 Degrees, the Complement of Latitude, and the Angle *d P c*, 38 Degrees, the Difference of Longitude, to find the Arch *d c*; but in order to Calculation, let fall the Perpendicular *P w*, then in the right-angled Triangle *P w c*, right-angled at *w*, you have given *P c* the Complement of Latitude 50 Degrees, and the Angle *w P c*, 19, half the Difference of Longitude, to find *w c* half the Diſtance in the Arch, which is found by the following Proportion, Caſe 2.

As Radius,
To Sine Complement of Latitude *P c*.
So Sine $\frac{1}{2}$ Difference of Longitude in Minutes, *w P c*.
To Sine $\frac{1}{2}$ the Diſtance in Miles. *w c* which doubled is the Diſtance requir'd.

For the Diſtance in the Parallel the Proportion, is
As Radius,
To Sine Comp. Latitude.
So Diff. Longitude in Minutes,
To the Diſtance in Miles in the Parallel,
Always more than the Diſtance in the Arch.

EXAM-

EXAMPLE 2.

If the Places differ both in Longitude and Latitude, as if one Place be in Latitude 40 North, and the other in Latitude 30 South, and differ in Longitude 58 Degrees; I desire to know their Distance.

P C Q draw *m b c*, the Parallel of 40 North, and *x h n*, the Parallel of 30 South; assume a Point as in *d*, for one Place, and draw the Meridian *P d s*, and from the Place where it cuts the Equinoctial, as in *e*, set off the Difference of Longitude 58, from *v* to *y*, and draw the Meridian *P y &*, and observe where it cuts the Parallel *x h z*, which is in *h*, which represents the other Place; then through *d* and *h*, draw the Circle *i d h k*; the Arch *d h* is the Distance requir'd.

CALCULATION.

In the oblique Triangle *d P h*, you have given *d P*, the Complement of one Latitude 50 Degrees, and *P h* the other Latitude added to 90, *viz.* 120 Degrees, and the Angle at *P* 58 Degrees; the Difference of Longitude to find *d h* by Case the Eighth, of oblique spherical Triangles.

And thus by a right Understanding, and true Application of spherical Triangles, all the Cases in Geography, with respect to Distance of Places, may be solved; which is so plain, that I need not enlarge upon it.

I might have added ſomething with reſpect to great Circle Sailing; but I ſhall not meddle with any Part of Navigation in this Treatiſe; becauſe I ſhall (God willing) ſhortly publiſh a Book of Navigation, and the Uſe of the new Globular Projection of Sea Charts, in which great Circle Sailing is render'd eaſy and practicable, although never accounted ſo upon any other Projection; that impartial thinking Men may be convinc'd, Whether a fictitious cylindrical Projection, grounded upon a Suppoſition known to be falſe, or a true Globular Projection, grounded upon, and deduc'd from ſound and real Principles, be moſt applicable to Practice, and fitteſt to be depended upon for Truth and Certainty.

APPEN-

APPENDIX.

Containing Tables of Right Aſcenſion, and of Oblique Aſcenſion, and Deſcenſion; for finding the Riſing and Setting of the Sun, with the Riſing, Southing, and Setting of the Moon, and the Southing of fixed Stars.

A Table of Right Aſcenſion.

D.	♈ H.	M.	♉ H.	M.	♊ H.	M.	♋ H.	M.	♌ H.	M.	♍ H.	M.
☉	0	0	1	52	3	51	6	0	8	9	10	8
1	0	4	1	55	3	55	6	4	8	12	10	12
2	0	7	1	59	4	0	6	9	8	17	10	16
3	0	11	2	3	4	4	6	13	8	21	10	20
4	0	15	2	7	4	8	6	17	8	25	10	24
5	0	18	2	12	4	12	6	22	8	29	10	27
6	0	22	2	15	4	16	6	26	8	34	10	31
7	0	26	2	19	4	21	6	31	8	38	10	35
8	0	29	2	22	4	25	6	35	8	42	10	39
9	0	33	2	26	4	29	6	39	8	46	10	42
10	0	37	2	30	4	33	6	43	8	50	10	46
11	0	40	2	34	4	38	6	48	8	54	10	50
12	0	44	2	38	4	42	6	52	8	58	10	54
13	0	48	2	42	4	46	6	57	9	2	10	57
14	0	51	2	46	4	51	7	1	9	6	11	1
15	0	55	2	50	4	55	7	5	9	10	11	5
16	0	59	2	54	4	59	7	9	9	14	11	9
17	1	3	2	58	5	3	7	14	9	18	11	12
18	1	6	3	2	5	8	7	18	9	22	11	16
19	1	10	3	6	5	12	7	22	9	26	11	20
20	1	14	3	10	5	16	7	27	9	30	11	23
21	1	18	3	14	5	21	7	31	9	34	11	27
22	1	21	3	18	5	25	7	35	9	38	11	31
23	1	25	3	22	5	29	7	39	9	41	11	34
24	1	29	3	26	5	34	7	44	9	45	11	38
25	1	33	3	31	5	38	7	48	9	49	11	42
26	1	36	3	35	5	43	7	52	9	53	11	45
27	1	40	3	39	5	47	7	56	9	57	11	49
28	1	44	3	43	5	51	8	0	10	1	11	53
29	1	48	3	47	5	56	8	5	10	5	11	56
30	1	52	3	51	6	0	8	9	10	8	12	0

A Table of Right Ascension.

D.	♎ H.	M.	♏ H.	M.	♐ H.	M.	♑ H.	M.	♒ H.	M.	♓ H.	M.
0	12	0	13	52	15	51	18	0	20	9	22	8
1	12	4	13	55	15	55	18	4	20	13	22	12
2	12	7	13	59	16	0	18	9	20	17	22	16
3	12	11	14	3	16	4	18	13	20	21	22	20
4	12	15	14	7	16	8	18	17	20	25	22	24
5	12	18	14	11	16	12	18	22	20	29	22	27
6	12	22	14	15	16	16	18	26	20	34	22	31
7	12	26	14	19	16	21	18	31	20	38	22	35
8	12	29	14	22	16	25	18	35	20	42	22	39
9	12	33	14	26	16	29	18	39	20	46	22	42
10	12	37	14	30	16	33	18	44	20	50	22	46
11	12	40	14	34	16	38	18	48	20	54	22	50
12	12	44	14	38	16	42	18	52	20	58	22	54
13	12	48	14	42	16	46	18	57	21	2	22	57
14	12	51	14	46	16	51	19	1	21	6	23	1
15	12	55	14	50	16	55	19	5	21	10	23	5
16	12	59	14	54	16	59	19	9	21	14	23	9
17	13	3	14	58	17	3	19	13	21	18	23	12
18	13	6	15	2	17	8	19	18	21	22	23	16
19	13	10	15	6	17	12	19	22	21	26	23	20
20	13	14	15	10	17	16	19	27	21	30	23	23
21	13	18	15	14	17	21	19	31	21	34	23	27
22	13	21	15	18	17	25	19	35	21	38	23	31
23	13	25	15	22	17	29	19	39	21	41	23	34
24	13	29	15	26	17	34	19	44	21	45	23	38
25	13	33	15	31	17	38	19	48	21	49	23	42
26	13	36	15	35	17	43	19	52	21	53	23	45
27	13	40	15	39	17	47	19	56	21	57	23	49
28	13	44	15	43	17	51	20	0	22	1	23	53
29	13	48	15	47	17	56	20	5	22	5	23	56
30	13	51	15	51	18	0	20	9	22	8	24	0

A Table of Oblique Ascension.

D.	♈ H.	M.	♉ H.	M.	♊ H.	M.	♋ H.	M.	♌ H.	M.	♍ H.	M.
0	18	0	18	52	20	0	21	47	0	18	3	9
1	18	2	18	54	20	3	21	51	0	23	3	14
2	18	3	18	56	20	6	21	56	0	29	3	20
3	18	5	18	58	20	9	22	1	0	35	3	26
4	18	7	19	0	20	12	22	5	0	40	3	32
5	18	9	19	2	20	15	22	9	0	46	3	38
6	18	10	19	4	20	18	22	14	0	51	3	43
7	18	12	19	6	20	21	22	19	0	57	3	49
8	18	13	19	8	20	24	22	24	1	2	3	55
9	18	15	19	10	20	27	22	28	1	8	4	1
10	18	16	19	13	20	30	22	33	1	14	4	7
11	18	18	19	15	20	34	22	39	1	20	4	12
12	18	20	19	17	20	37	22	43	1	26	4	18
13	18	22	19	19	20	41	22	48	1	32	4	23
14	18	23	19	21	20	44	22	53	1	37	4	29
15	18	25	19	23	20	48	22	58	1	43	4	34
16	18	27	19	26	20	52	23	3	1	49	4	40
17	18	29	19	28	20	55	23	9	1	54	4	46
18	18	31	19	30	20	59	23	14	2	0	4	52
19	18	32	19	32	21	2	23	19	2	6	4	58
20	18	34	19	35	21	6	23	24	2	12	5	4
21	18	36	19	37	21	10	23	29	2	18	5	9
22	18	37	19	39	21	14	23	34	2	23	5	15
23	18	39	19	42	21	18	23	40	2	29	5	21
24	18	41	19	45	21	22	23	45	2	34	5	26
25	18	43	19	48	21	26	23	51	2	40	5	32
26	18	45	19	50	21	30	23	56	2	46	5	37
27	18	47	19	53	21	34	0	2	2	52	5	43
28	18	48	19	55	21	39	0	7	2	58	5	48
29	18	50	19	58	21	43	0	13	3	3	5	54

A Table of Oblique Aſcenſion.

D.	♎ H.	M.	♏ H.	M.	♐ H.	M.	♑ H.	M.	♒ H.	M.	♓ H.	M.
0	6	0	8	51	11	43	14	13	16	0	17	8
1	6	5	8	57	11	48	14	17	16	3	17	10
2	6	11	9	2	11	53	14	21	16	6	17	12
3	6	17	9	8	11	58	14	25	16	8	17	14
4	6	23	9	14	12	4	14	30	16	10	17	15
5	6	28	9	20	12	9	14	34	16	13	17	17
6	6	34	9	26	12	15	14	38	16	15	17	19
7	6	40	9	32	12	20	14	42	16	18	17	21
8	6	46	9	37	12	25	14	46	16	21	17	22
9	6	51	9	43	12	30	14	50	16	23	17	24
10	6	57	9	48	12	36	14	54	16	25	17	26
11	7	3	9	54	12	41	14	58	16	28	17	28
12	7	8	10	0	12	46	15	1	16	30	17	29
13	7	14	10	6	12	51	15	5	16	32	17	31
14	7	20	10	11	12	56	15	9	16	35	17	33
15	7	26	10	17	13	2	15	12	16	37	17	34
16	7	31	10	23	13	7	15	16	16	39	17	36
17	7	37	10	28	13	12	15	19	16	41	17	38
18	7	43	10	34	13	17	15	22	16	43	17	40
19	7	48	10	40	13	22	15	26	16	45	17	42
20	7	54	10	46	13	27	15	30	16	48	17	44
21	7	59	10	51	13	32	15	33	16	50	17	45
22	8	6	10	57	13	36	15	36	16	52	17	47
23	8	11	11	3	13	41	15	39	16	54	17	49
24	8	17	11	9	13	46	15	42	16	56	17	50
25	8	22	11	14	13	51	15	45	16	58	17	52
26	8	28	11	20	13	55	15	48	17	0	17	53
27	8	34	11	26	13	59	15	51	17	2	17	55
28	8	40	11	31	14	4	15	54	17	4	17	57
29	8	46	11	37	14	9	15	57	17	6	17	59

A Table of Oblique Descension.

D.	♈ H.	M.	♉ H.	M.	♊ H.	M.	♋ H.	M.	♌ H.	M.	♍ H.	M.
0	6	0	8	51	11	43	14	13	16	0	17	8
1	6	5	8	57	11	48	14	17	16	3	17	10
2	6	11	9	2	11	53	14	21	16	6	17	12
3	6	17	9	8	11	58	14	25	16	8	17	14
4	6	23	9	14	12	4	14	30	16	10	17	15
5	6	28	9	20	12	9	14	34	16	13	17	17
6	6	34	9	26	12	15	14	38	16	15	17	19
7	6	40	9	32	12	20	14	42	16	18	17	21
8	6	46	9	37	12	25	14	46	16	21	17	22
9	6	51	9	43	12	30	14	50	16	23	17	24
10	6	57	9	48	12	36	14	54	16	25	17	26
11	7	3	9	54	12	41	14	58	16	28	17	28
12	7	8	10	0	12	46	15	1	16	30	17	29
13	7	14	10	6	12	51	15	5	16	32	17	31
14	7	20	10	11	12	56	15	9	16	35	17	33
15	7	26	10	17	13	2	15	12	16	37	17	34
16	7	31	10	23	13	7	15	16	16	39	17	36
17	7	37	10	28	13	12	15	19	16	41	17	38
18	7	43	10	34	13	17	15	22	16	43	17	40
19	7	48	10	40	13	22	15	26	16	45	17	42
20	7	54	10	46	13	27	15	30	16	48	17	44
21	7	59	10	51	13	32	15	33	16	50	17	45
22	8	6	10	57	13	36	15	36	16	52	17	47
23	8	11	11	3	13	41	15	39	16	54	17	49
24	8	17	11	9	13	46	15	42	16	56	17	50
25	8	22	11	14	13	51	15	45	16	58	17	52
26	8	28	11	20	13	55	15	48	17	0	17	53
27	8	34	11	26	13	59	15	51	17	2	17	55
28	8	40	11	31	14	4	15	54	17	4	17	57
29	8	46	11	37	14	9	15	57	17	6	17	59

A Table of Oblique Descension.

D.	♎ H.	M.	♏ H.	M.	♐ H.	M.	♑ M.	M.	♒ H.	M.	♓ H.	M.
0	18	0	18	52	20	0	21	47	0	18	3	9
1	18	2	18	54	20	3	21	51	0	23	3	14
2	18	3	18	56	20	6	21	56	0	29	3	20
3	18	5	18	58	20	9	22	1	0	35	3	26
4	18	7	19	0	20	12	22	5	0	40	3	32
5	18	9	19	2	20	15	22	9	0	46	3	38
6	18	10	19	4	20	18	22	14	0	51	3	43
7	18	12	19	6	20	21	22	19	0	57	3	49
8	18	13	19	8	20	24	22	24	1	2	3	55
9	18	15	19	10	20	27	22	28	1	8	4	1
10	18	16	19	13	20	30	22	33	1	14	4	7
11	18	18	19	15	20	34	22	39	1	20	4	12
12	18	20	19	17	20	37	22	43	1	26	4	18
13	18	22	19	19	20	41	22	48	1	32	4	23
14	18	23	19	21	20	44	22	53	1	37	4	29
15	18	25	19	23	20	48	22	58	1	43	4	34
16	18	27	19	26	20	52	23	3	1	49	4	40
17	18	29	19	28	20	55	23	9	1	54	4	46
18	18	31	19	30	20	59	23	14	2	0	4	52
19	18	32	19	32	21	2	23	19	2	6	4	58
20	18	34	19	35	21	6	23	24	2	12	5	4
21	18	36	19	37	21	10	23	29	2	18	5	9
22	18	37	19	39	21	14	23	34	2	23	5	15
23	18	39	19	42	21	18	23	40	2	29	5	21
24	18	41	19	45	21	22	23	45	2	34	5	26
25	18	43	19	48	21	26	23	51	2	40	5	32
26	18	45	19	50	21	30	23	56	2	46	5	37
27	18	47	19	53	21	34	0	2	2	52	5	43
28	18	48	19	55	21	39	0	7	2	58	5	48
29	18	50	19	58	21	43	0	13	3	3	5	54

A Table of the Moon's Motion.

Hours.	12 Deg. d.	m.	13 Deg. d.	m.	14 Deg. d.	m.	15 Deg. d.	m.	16 Deg. .	m.
1	0	30	0	32	0	35	0	37	0	40
2	1	0	1	5	1	10	1	15	1	20
3	1	30	1	37	1	45	1	52	2	0
4	2	0	2	10	2	20	2	30	2	40
5	2	30	2	43	2	35	3	7	3	20
6	3	0	3	15	3	30	3	45	4	0
7	3	30	3	48	4	5	4	22	4	40
8	4	0	4	20	4	40	5	0	5	20
9	4	30	4	53	5	15	5	37	6	0
10	5	0	5	25	5	50	6	15	6	40
11	5	30	5	58	6	25	6	52	7	20
12	6	0	6	30	7	0	7	30	8	0
13	6	30	7	3	7	35	8	7	8	40
14	7	0	7	35	8	10	8	45	9	20
15	7	30	8	8	8	45	9	22	10	0
16	8	0	8	40	9	20	10	0	10	40
17	8	30	9	13	9	55	10	37	11	20
18	9	0	9	45	10	30	11	15	12	0
19	9	30	10	18	11	5	11	52	12	40
20	10	0	10	50	11	40	12	30	13	20
21	10	30	11	23	12	15	13	7	14	0
22	11	0	11	55	12	50	13	45	14	40
23	11	30	12	28	13	25	14	22	15	20
24	12	0	13	0	14	0	15	0	16	0

☽ Ag.	☽ So. H.	M.
1	0 A	48
2	1	36
3	2	24
4	3	12
5	4	0
6	4	48
7	5	36
8	6	24
9	7	12
10	8	0
11	8	48
12	9	36
13	10	24
14	11	12
15	12	0
16	0 M	48
17	1	36
18	2	24
19	3	12
20	4	0
21	4	48
22	5	36
23	6	24
24	7	12
25	8	0
26	8	48
27	9	36
28	10	24
29	11	12
30	12	0

THERE is in all Caſes two Numbers to be ſubſtracted from each other, the Remainder is the Anſwer to the Queſtion, in Hours and Minutes: Of theſe two Numbers the Sun's right Aſcenſion is always one, becauſe we begin to reckon the Aſtronomical Hours at Noon; then for Riſing, uſe oblique Aſcenſion; for Southing, uſe right Aſcenſion; and for Setting, uſe oblique Deſcenſion, *viz.* If you ſeek for the Sun's Riſing, ſubſtract his right Aſcenſion from his oblique Aſcenſion; the Remainder is the Time of Sun's Riſing; but if you ſeek the Time of Sun's Setting, ſubſtract his right Aſcenſion, from his oblique Deſcenſion, the Remainder is the Time of Sun's Setting. Again, If you deſire to know the Time of the Moon's Coming to South, ſubſtract the Sun's right Aſcenſion from the Moon's right Aſcenſion, the Remainder is the Time of Moon's Southing; but if you would know the Time of the Moon's Setting, ſubſtract the Sun's right Aſcenſion, from the Moon's oblique Deſcenſion, the Remainder is the Time of Moon's Setting, borrowing 24 Hours if Subſtraction cannot otherwiſe be made, and if the Remainder, be more than 12, caſt away 12, and the Remainder is the Hour and Minute in the Morning; but if leſs than 12, it is the Hour and Minute in the Afternoon.

If you ſeek the Riſing, Southing, or Setting of the Moon, and would be exact; you muſt firſt compute in your Mind about what Time it will happen, and adjuſt her Place to that Hour, and then work as before.

EX-

EXAMPLE.

I desire to know the Time of Sun's Rising, *October.* 19. 1720. I look in the *Ephemeris*, and find the Sun is in 7 D. 9. M. of ♏. I look for 7 D. (rejecting the Minutes) and find the oblique Ascension of 7 D. of ♏, is 9 H. 32 M. and the right Ascension thereof is 14 H. 19 M. which substracted (having first suppos'd 24 Hours to be added to the oblique Ascension, as before directed) the Remainder is 19 13; from which abating 12, the Remainder 7 H. 13 M. is the Time of Sun's Rising, *October* 19, as was requir'd.

9 32
14 19
19 13
12
7 13

EXAMPLE 2.

I desire to know the Time of Moon's Setting, *Nov.* 21. 1720.

If we take the Place of the Moon at Noon, and use it as her true Place, it would beget a small Error, but for more Exactness, consider that the Moon is 3 Days old, and therefore will not set before 6 or 7 a Clock; I therefore compute her Place for 6 a Clock, thus;

I find the Moon, *Nov.* 21, at Noon, is ♑ 16 D. 55 M. her Motion from that Day at Noon, till *Nov.* 22. at Noon, is about 15 Degrees; I therefore look in the fourth Column of the last Table, under 15 Degrees, and against 6 Hours, I find 3 D. 45 M. which is the Degrees and Minutes the Moon has gone since Noon, that added to her Place at Noon, ♑ 16 55, makes ♑ 20.40, which being near 21 Degrees, and not regarding odd Minutes in this Case, I seek the oblique Descension of ♑ 21, and find it is 23 D. 29 M. I also